

# Using optical speckle in multimode waveguides for compressive sensing

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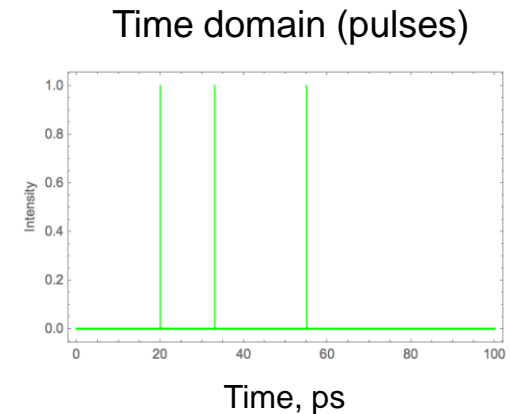
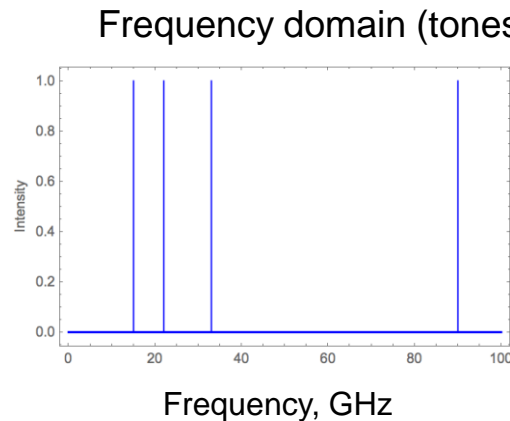
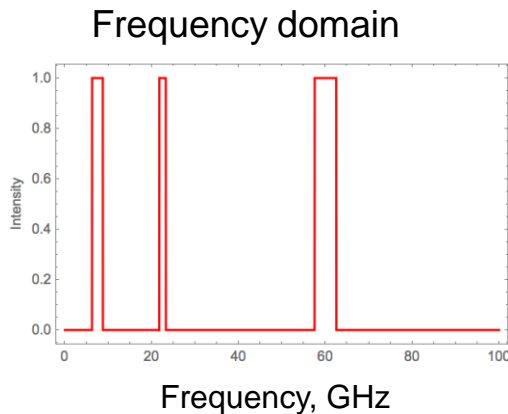
3 June 2016

# Outline

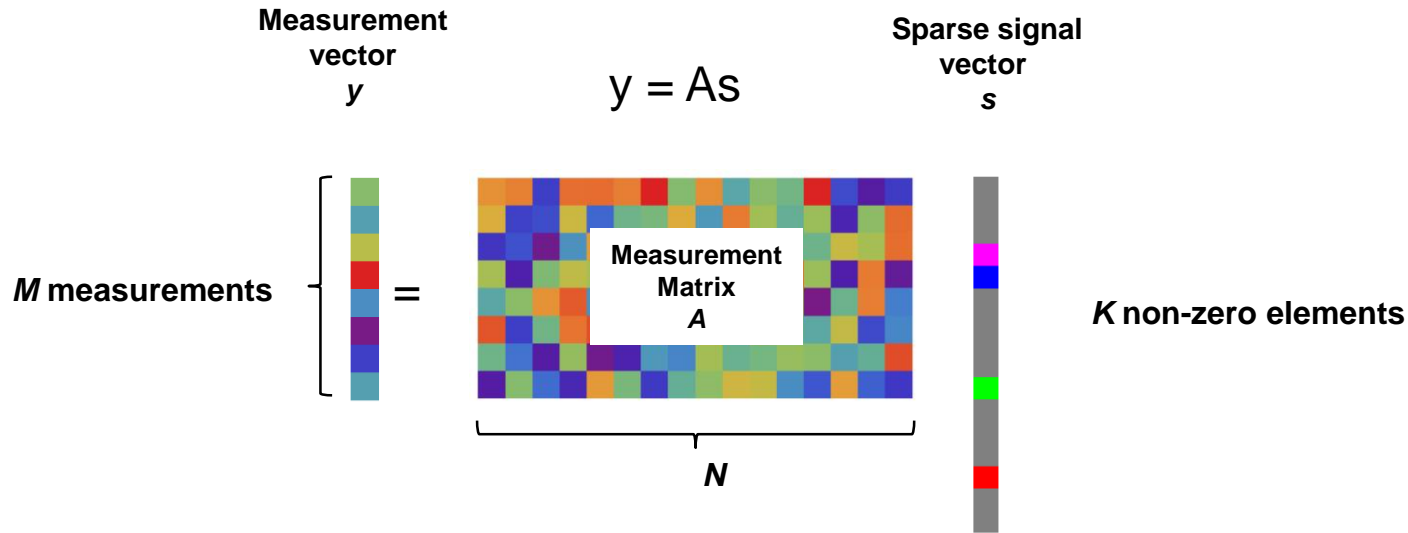
- Motivation for Compressive Sensing for GHz-band RF signals
- Compressive sensing
  - *Sparsity*
  - *Mixing down in dimension*
  - *Recovery*
- Electronic CS system
- Photonic CS systems
  - *Measurement matrix*
  - *Calibration*
  - *Path to photonic integrated circuit for CS*
- Photonic CS using speckle in multimode waveguide
- Other speckle-based photonic systems
- Conclusions

# Motivation for compressive sensing

- Nyquist-rate ADCs for GHz-band RF signals ...
  - *generate a tremendous amount of data*
    - *Rapidly fill storage buffers*
    - *Overwhelm processors*
    - *Swamp communication links*
  - *have limited performance*
  - *consume significant power*
- Signals of interest in GHz band are often sparse



# Compressive Sensing--pulses



- $N$  is # measurements needed to measure the signal at Nyquist rate
- $K$  is the sparsity (# of pulses, sinusoids,...)
- $M$  is the # CS measurements needed to recover the signal  $s$

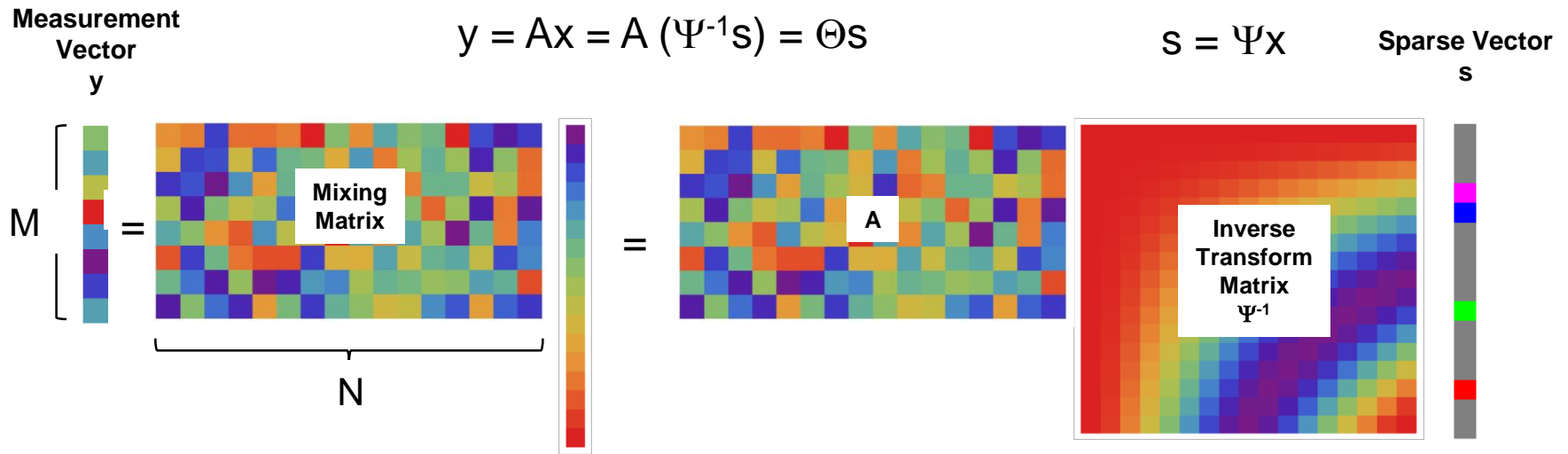
$$N \gg K \text{ and } M > K$$

- CS theorems show that for certain classes of Measurement Matrix  $A$ , one can recover  $s$  with high probability if

$$M > c K \log(N/K) \quad (c \sim O[1])$$

***Accurate knowledge of the Measurement Matrix  $A$  is critical for CS recovery calculations***

# Compressive Sensing—arbitrary waveforms



- $N$  is # measurements needed to measure the signal at Nyquist rate
- $K$  is the sparsity (# of pulses, sinusoids,...)
- $M$  is the # CS measurements needed to recover the signal  $s$

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**Accurate knowledge of the Measurement Matrix  $A$  is critical for CS recovery calculations**

# The 3 fundamental aspects of CS

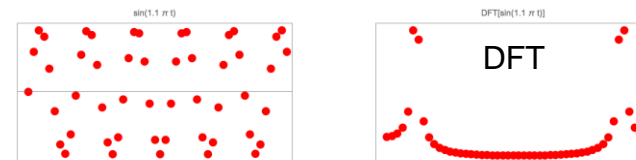
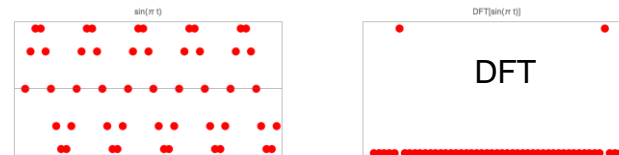
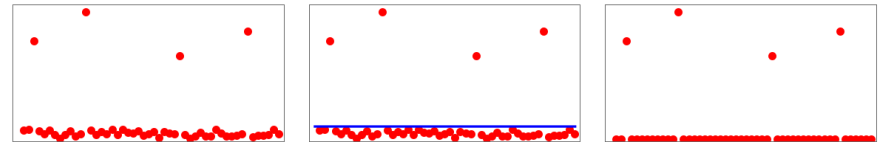
- Sparse signals/images
  - *Subtract off known background*
  - *Threshold noise*
  - *Transform to a basis in which signal is sparse*
- Analog Mixing
  - *Wideband converter—mix with pseudo-random waveforms (e.g. PRBS)*
  - *Single pixel camera—mix with pseudo-random images*
- Signal/Image Recovery
  - *Need accurate knowledge of analog measurement matrix*
  - *Exploit sparsity to limit solution space*
  - *Wide range of algorithms and codes now available*
    - Penalized  $\ell_1$ -norm
    - Orthogonal matching pursuit

# Sparsity

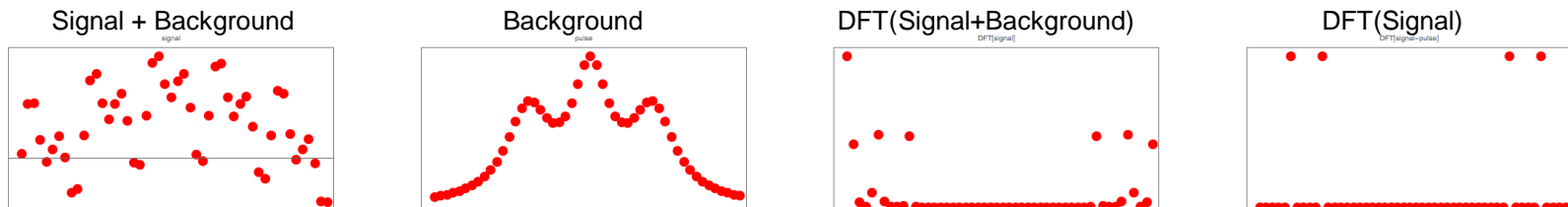
- A sparse vector/matrix—mostly 0's
  - What percentage is sparse?



- Noise—threshold signal
- Sine waves—Transform signal
  - $\sin(\pi t)$
- Beware of off-grid frequencies
  - $\sin(1.1 \pi t)$



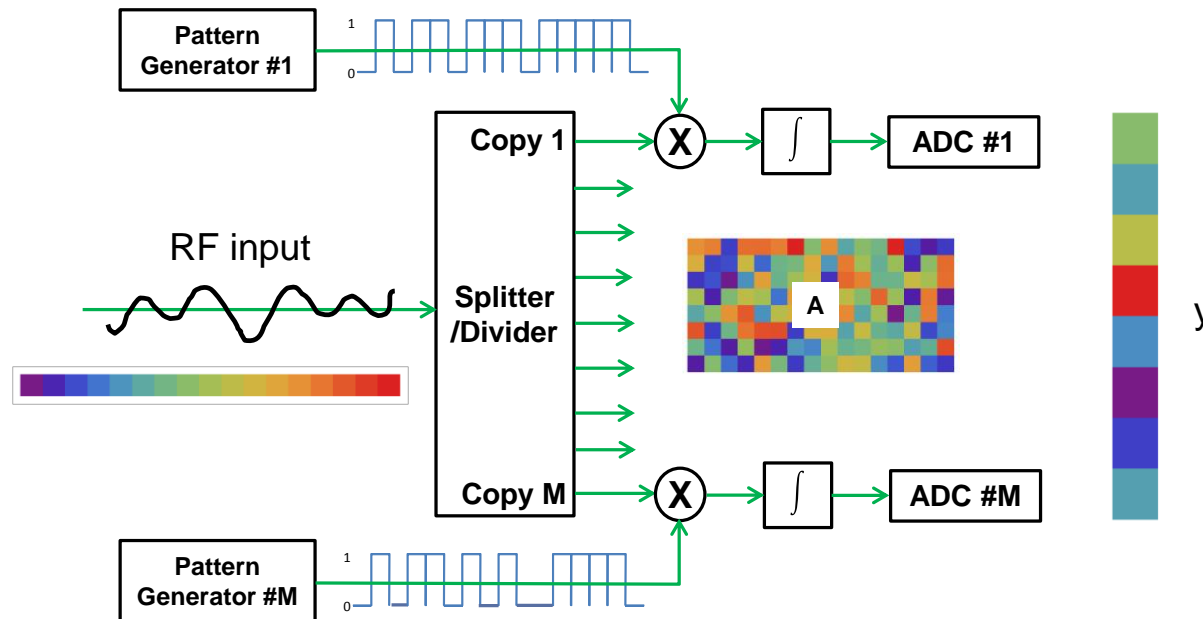
- Subtract off non-sparse background



*There is always a transform that makes a signal sparse—but you may not know it!*

# Electronic CS system for GHz-band RF Signals

- Split signal into  $M$  copies
- Multiply each copy by different pseudo-random bit sequence (PRBS) of length  $N$
- Integrate for duration of PRBS, sample, and digitize
- Issues:
  - Requires  $M$  electronic pattern generators
  - Size, weight, and power
  - Random noise and jitter within PRBS limit recovery



Mishali and Eldar, *IEEE Journal of Selected Topics in Signal Processing*, Vol. 4, pp. 375 (2010).



# Photonic undersampling and compressive sensing Origins

Before 2008

- Photonic ADC work
- Photonic down-conversion/down-sampling

2008

- Moshe Horowitz' group Technion: Multi-rate asynchronous sampling
- Johns Hopkins U. Appl. Phys. Lab: Non-uniform sampling

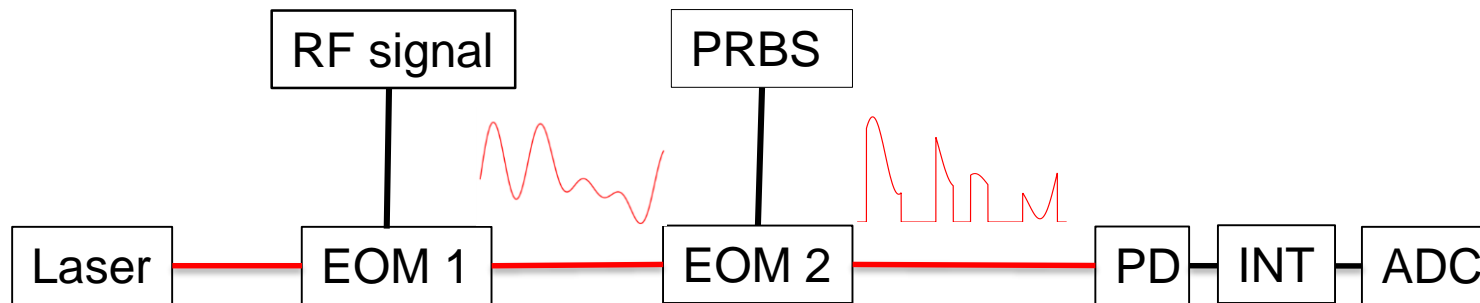
2010 Photonic CS

- Technion group use CS techniques to recover signals
- JH APL group recognizes non-uniform sampling is a form of CS
- Aerospace group proposes parallel multi-rate sampling CS

2011-2016

Approximately 30 papers on Photonic CS

# Serial CS system using PRBS and EOMs



Nichols and Bucholtz 2011

Chi et al. 2012

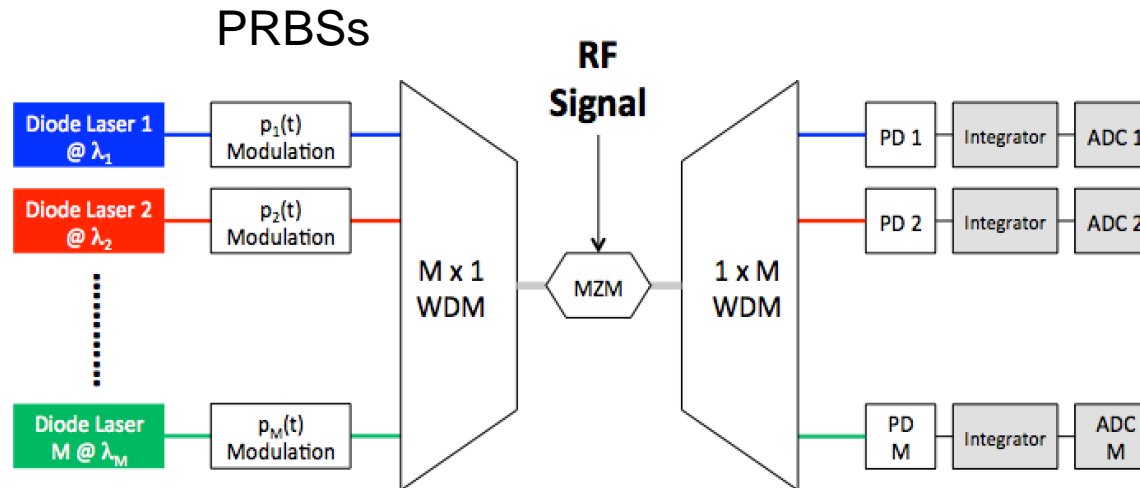
Yan *et al.* 2012

McKenna *et al.* 2013

Chen et al. 2013

Yin *et al.* 2013

# Parallel CS system using PRBSs and EOM

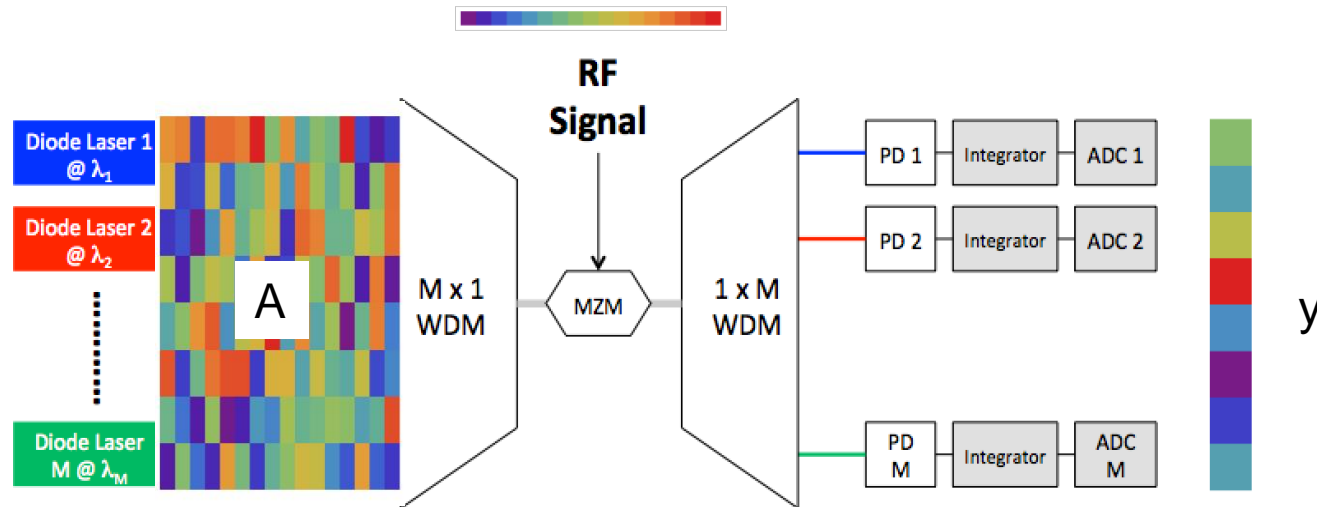


Proposed WDM pair to obtain simultaneous measurement of all elements in measurement vector  $\mathbf{y}$  (Nan *et al.* 2011)

Pseudo-random bit sequences impressed on cw diode lasers prior to RF signal

*Still has amplitude and timing jitter of electronics*

# Proposed Parallel CS system using PRBS and EOM



Proposed WDM pair to obtain simultaneous measurement of all elements in measurement vector  $y$  (Nan *et al.* 2011)

Pseudo-random bit sequences impressed on cw diode lasers prior to RF signal

*Still has amplitude and timing jitter of electronics*

# CW or pulsed laser?

## CW laser

Advantages: Simple, efficient, low average power

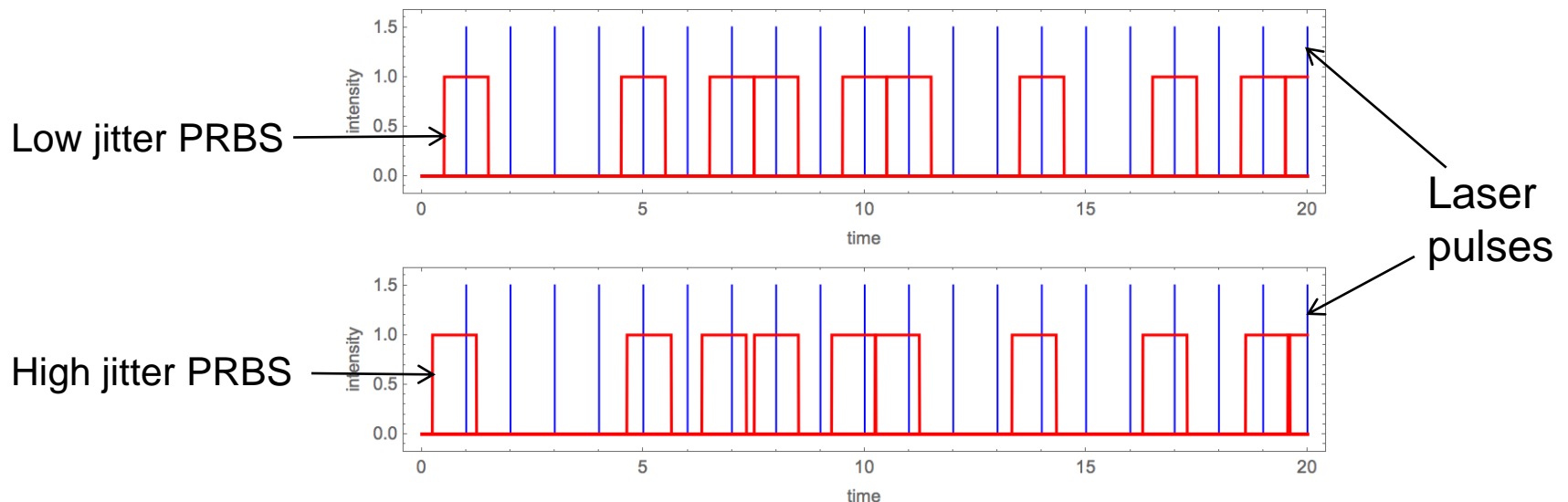
Disadvantage: Timing jitter of PRBS mapped onto optical intensity

## Pulsed laser

Advantage: Timing jitter of PRBS removed by low jitter mode-locked laser

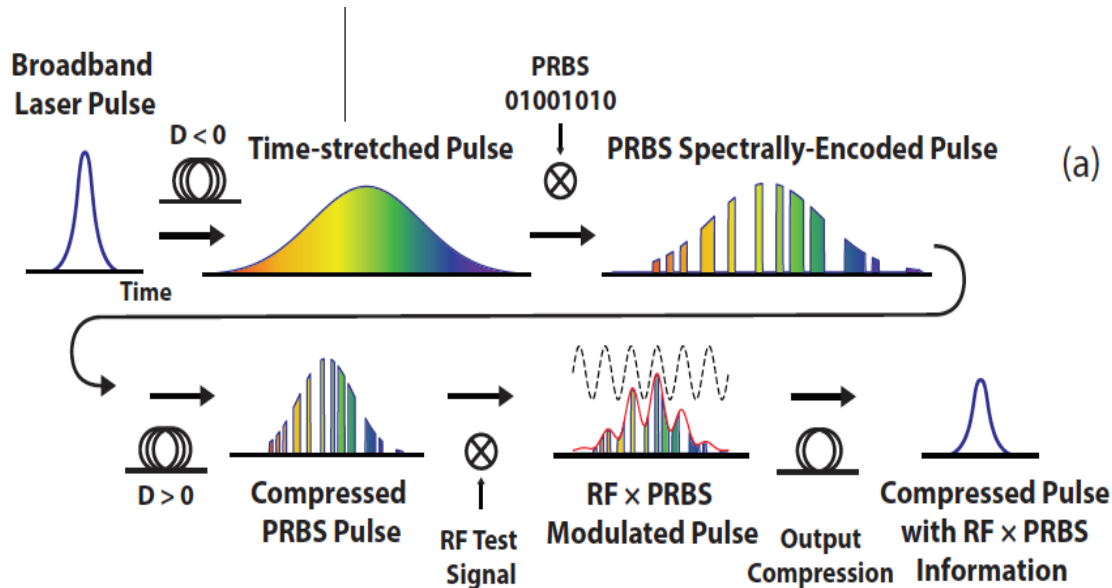
Disadvantages: high peak power on photodiode

Neither system avoids amplitude noise of PRBS generator



***Low-jitter pulsed laser removes effect of PRBS jitter***

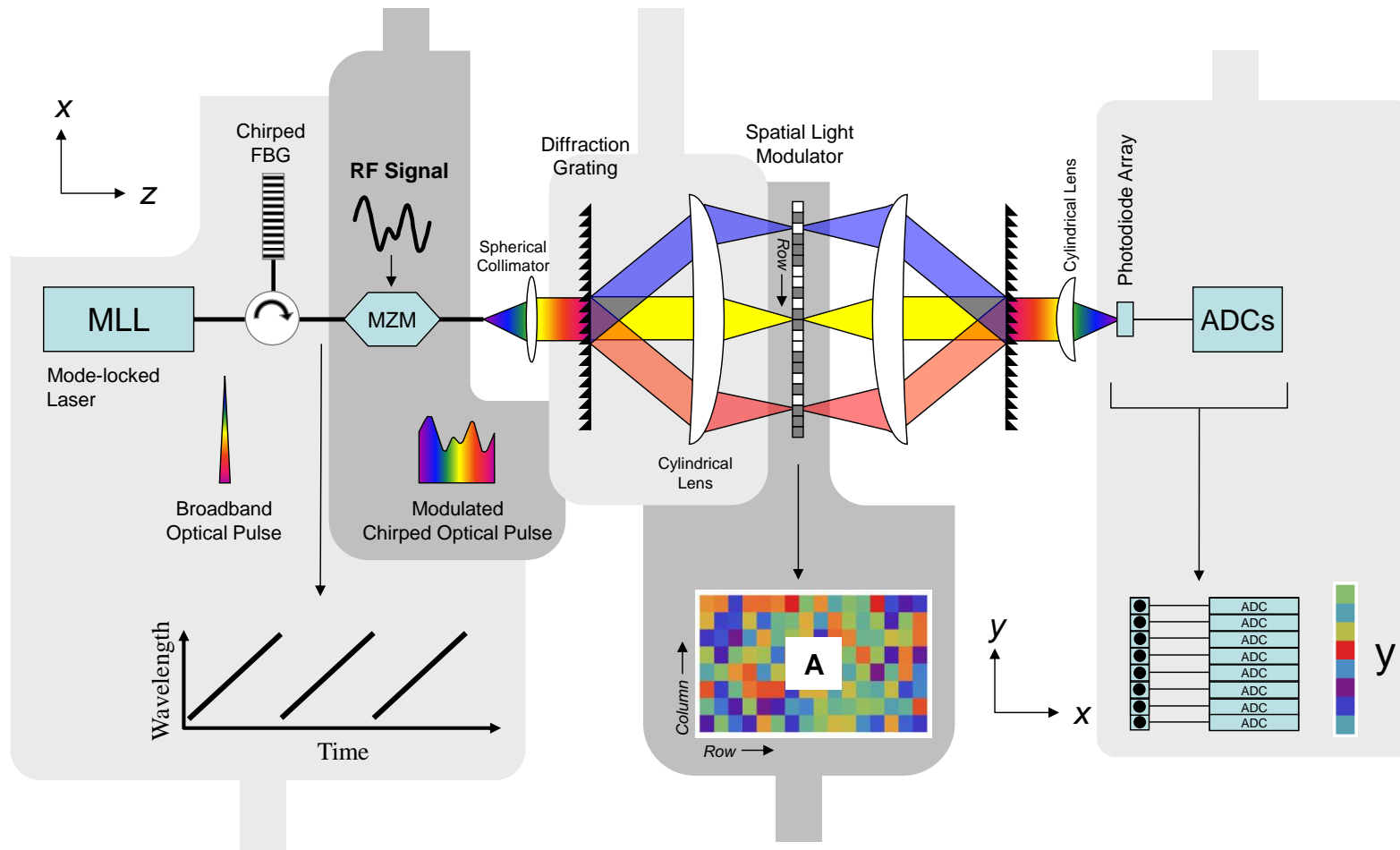
# Time stretching/compression to decrease effective amplitude and timing jitter in PRBS



Demonstrated use of stretching/compression to increase effective rate of PRBS  
(Bosworth and Foster Optics Letters 2013)

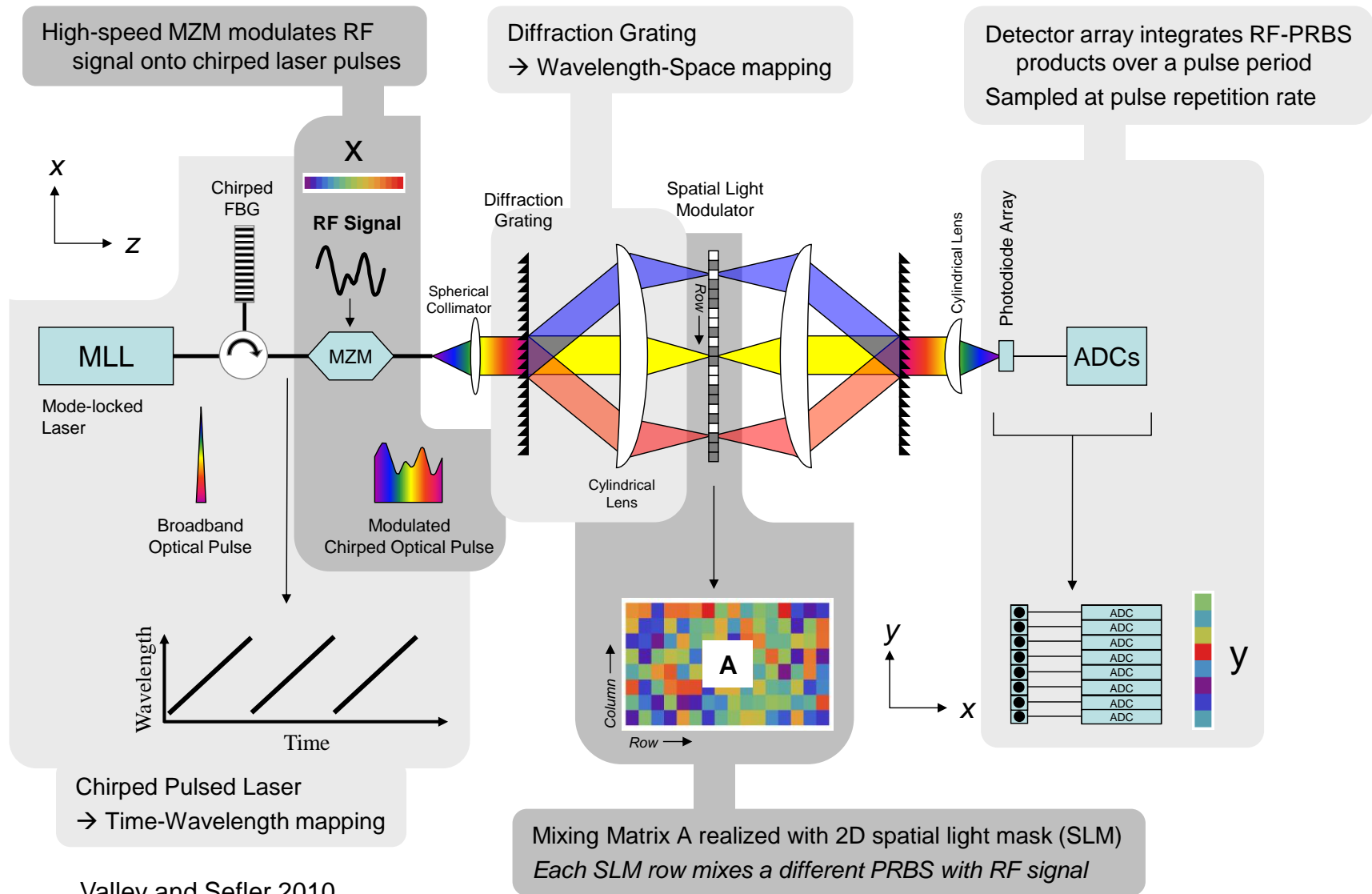
***Time-stretching allows use of lower rate PRBS with less jitter***

# Parallel CS system with PRBS addressing 2D Spatial Light Modulator



Valley and Sefler 2010

# Parallel CS system with PRBS addressing 2D Spatial Light Modulator

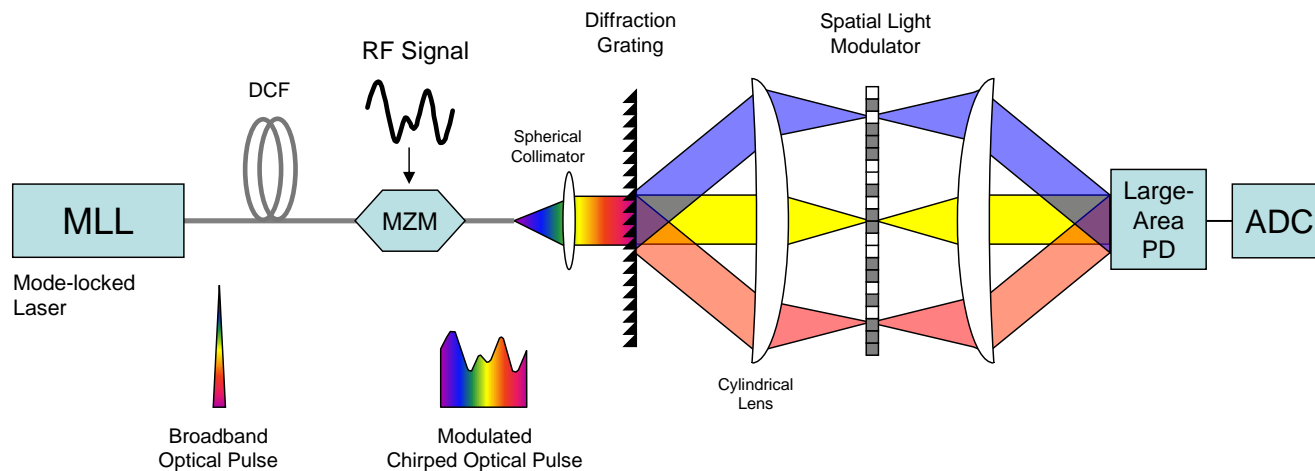


Valley and Sefler 2010

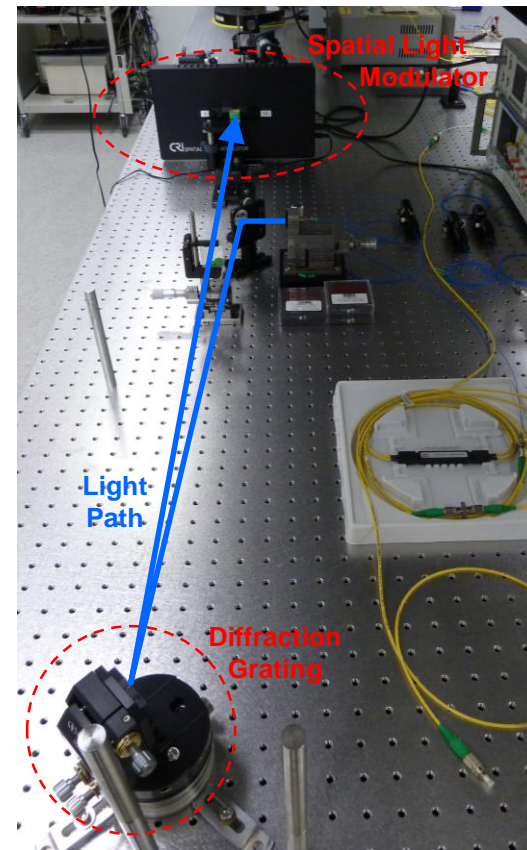


# Serial Experimental Demonstration with 1D SLM

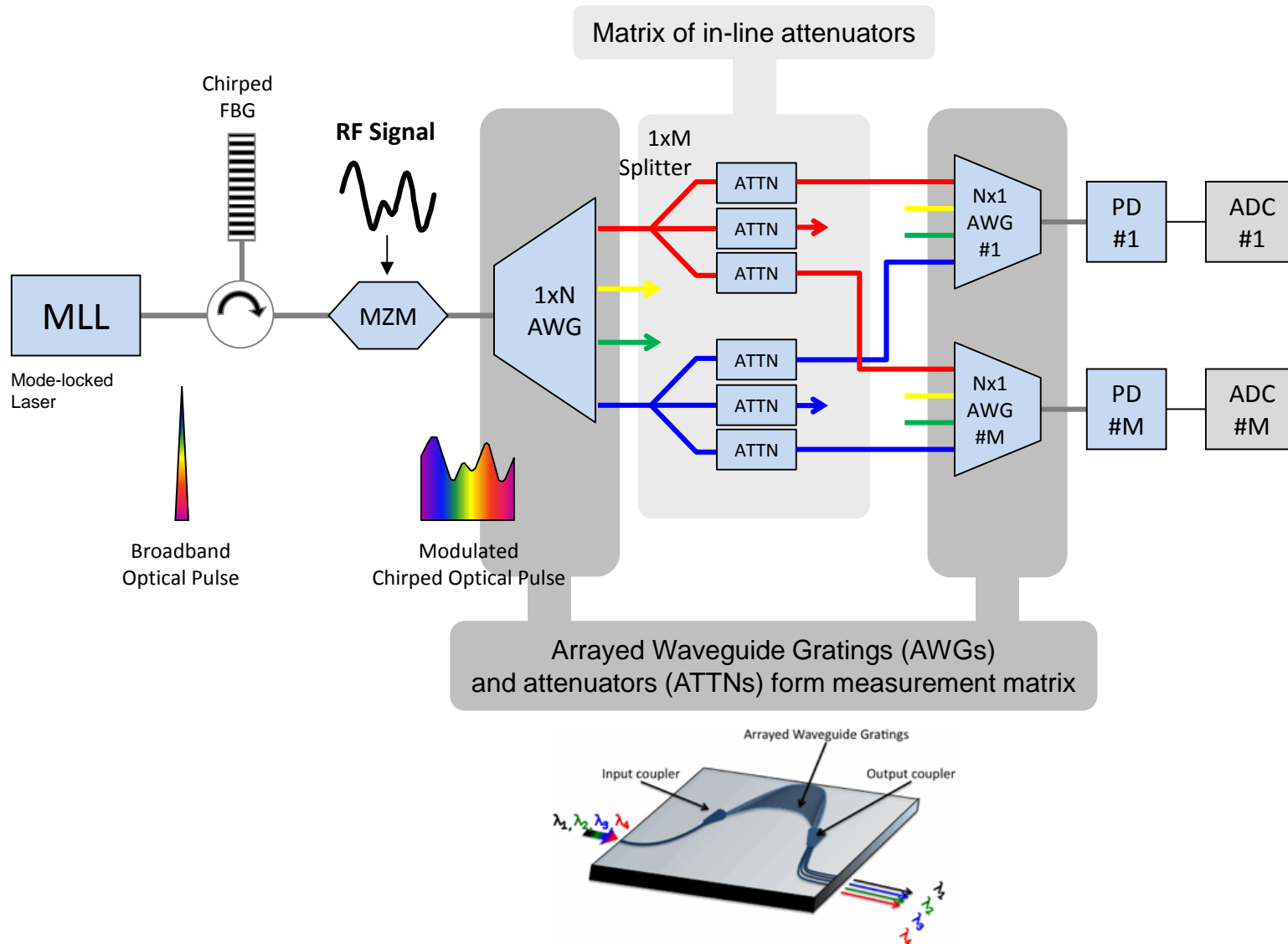
- Used 1D liquid-crystal SLM
- RF signals synchronized to laser pulse repetition rate
- Measurements made sequentially by stepping SLM through rows of A
- RF tones (sparse in frequency domain) and pulses (sparse in time domain) recovered using penalized  $\ell_1$ -norm



Valley, Seifler, and Shaw, 2012

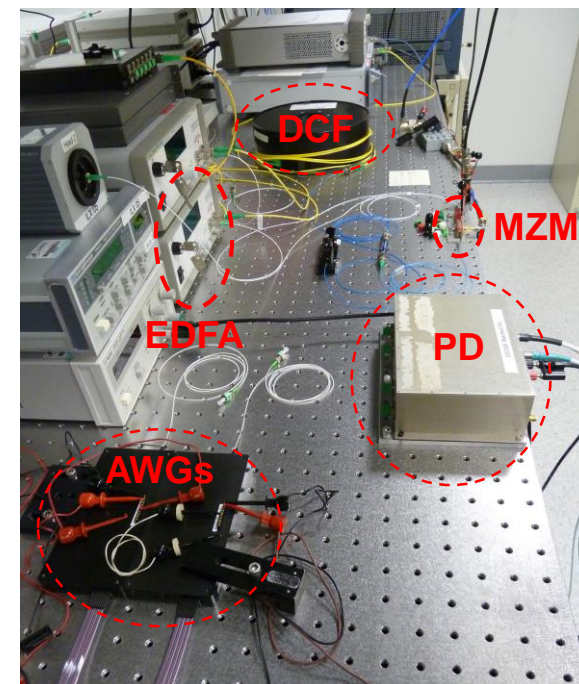
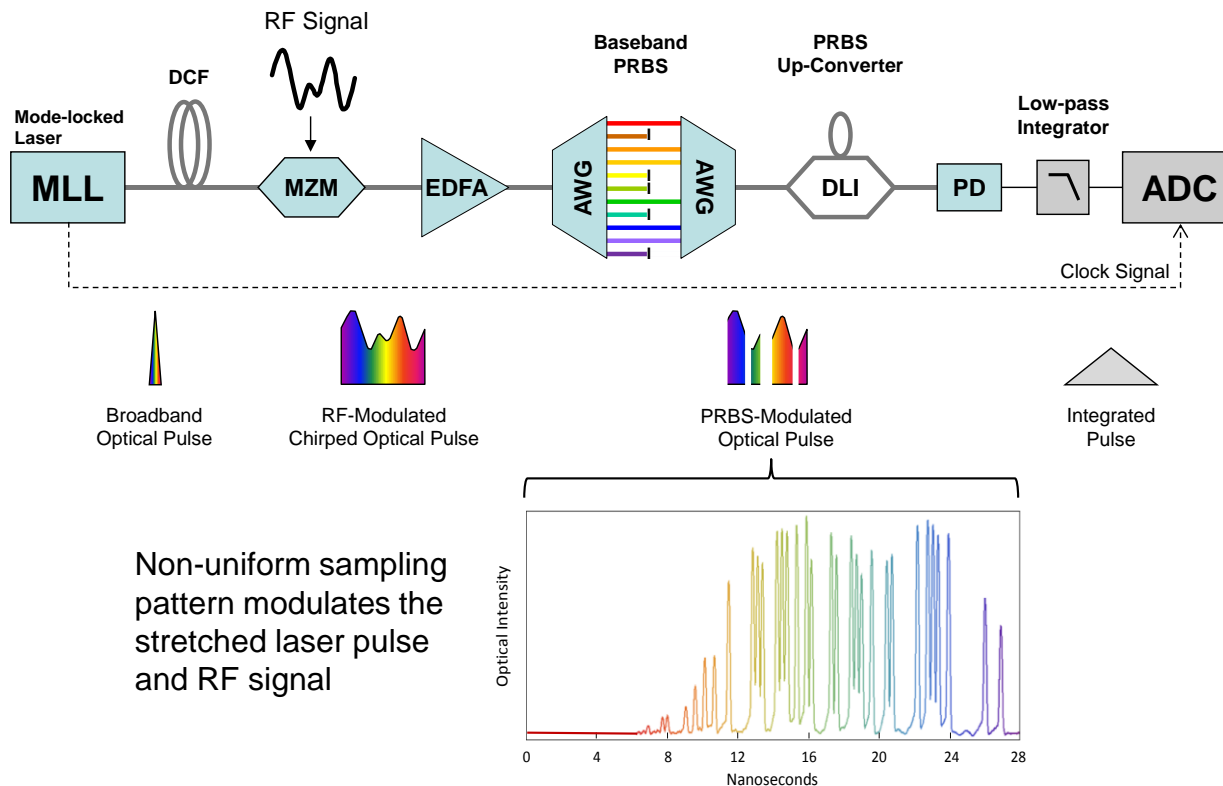


# Parallel CS system using WDMs (AWGs)



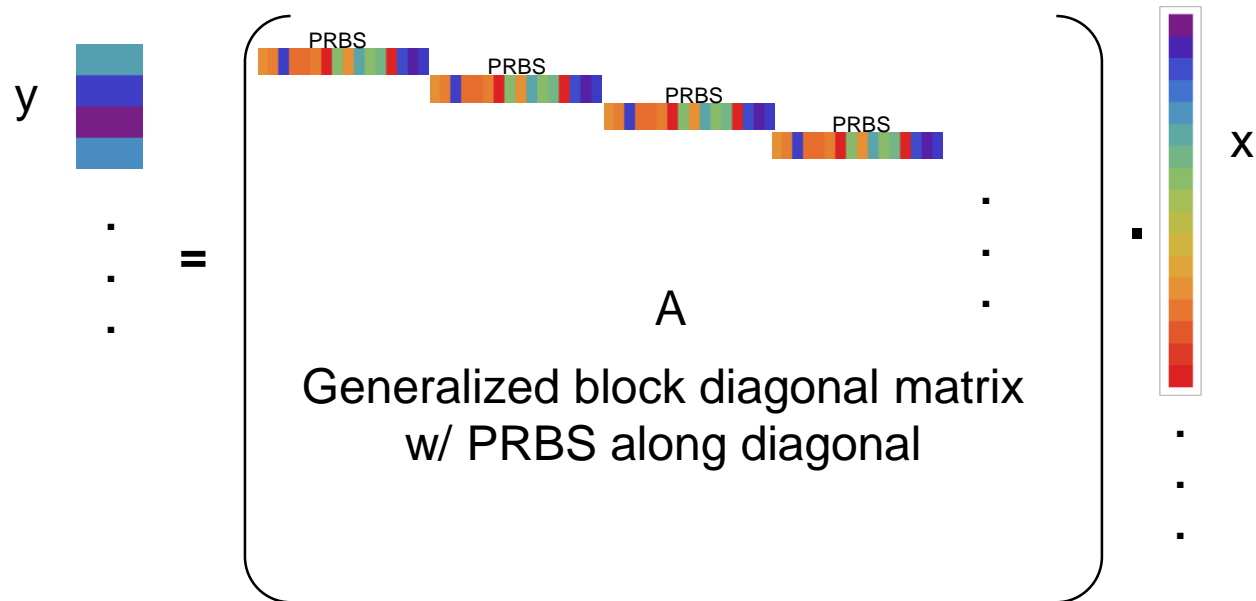
# Serial experimental demonstration using AWGs

- Single row of Mixing Matrix A
- AWG: 96 wavelength channels @ 0.4-nm channel spacing
  - Selected channels blocked to form PRBS
- Delay-line interferometer (DLI) Up-converter
- RF bandwidths from 4 to 20 GHz



# Periodic non-uniform sampling with integration

- Periodic non-uniform sampling and integration
  - *Single PRBS repeated*
    - *Measurement Matrix has generalized block diagonal structure*
    - *Arbitrary number of measurements  $y_i$*
    - *Arbitrary length of Signal  $x$*
- Effective sampling rate is PRBS/laser rep rate
- No RF-to-laser synchronization required
- Useful for long duration RF signals (e.g. chirped pulses)

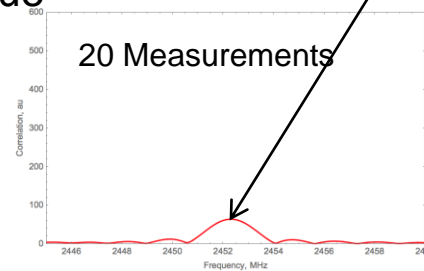
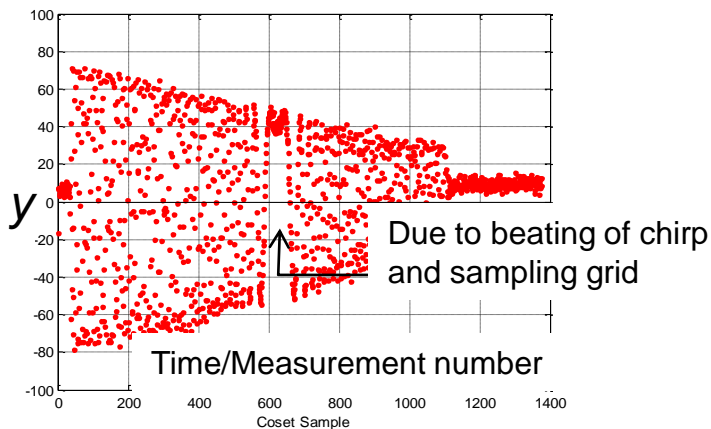


# Experimental results for RF chirped pulses

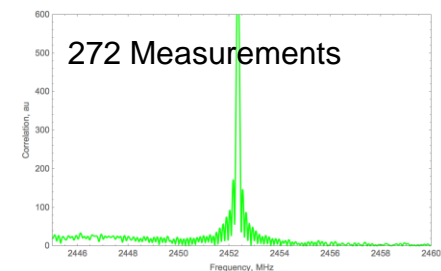
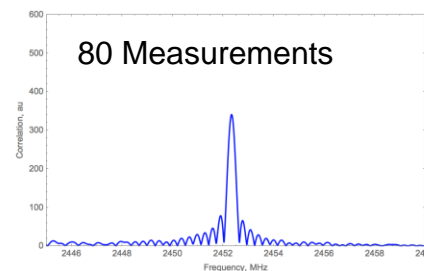
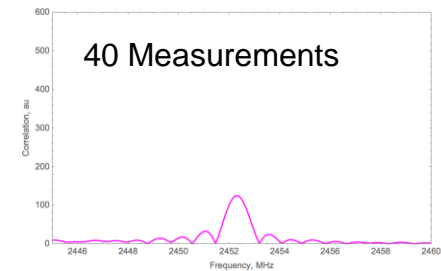
## Carrier Recovery vs. Number of Measurements

### WDM mixing

- RF Parameters:
  - Chirp = 20 MHz / 30  $\mu$ s
  - Carrier Freq = 2.453 GHz
- PRBS Rep Rate (effective sample rate) = 35 MHz
- Maximum likelihood recovery technique



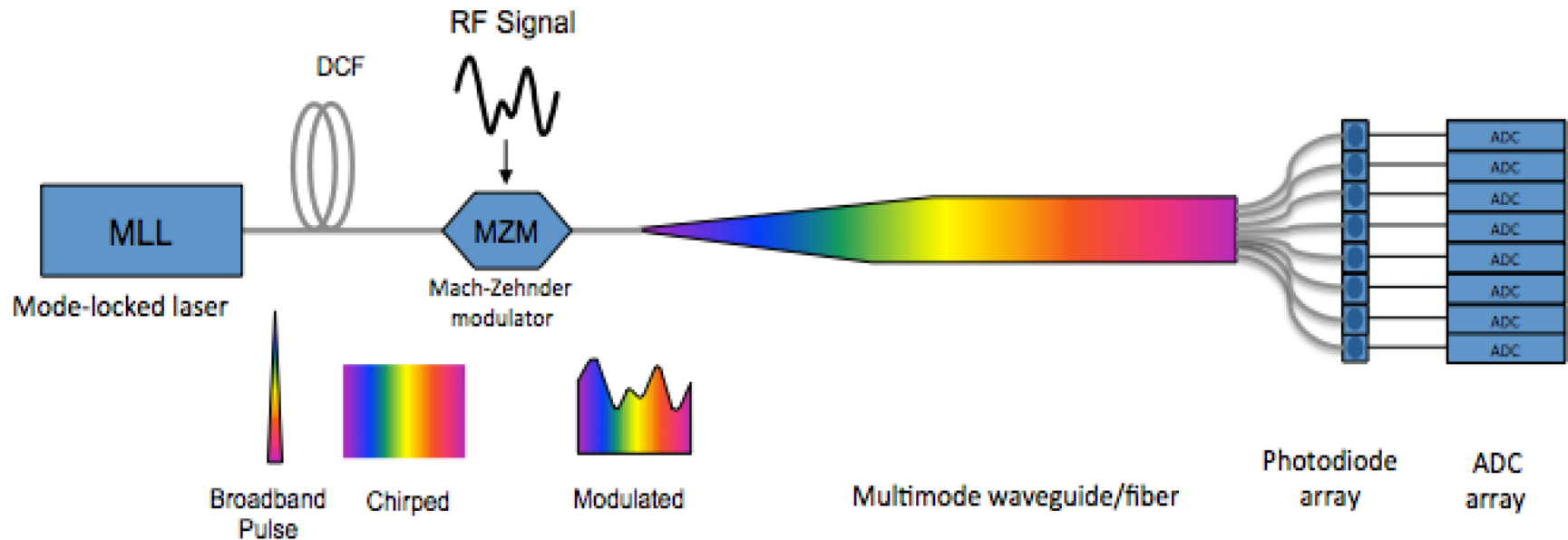
Location of peak  
gives the  
frequency



# Desirable Properties for Photonic CS System

- Components integrable in one or more photonic integrated circuits
  - Avoid free-space optics
  - Minimize fiber-coupled devices
- Static or low-error PRBS generation that can be calibrated
  - Spatial light modulators
  - Pulse or bandwidth compression (Bosworth and Foster 2013)
  - Optical pulse in the center of each PRBS bit (Chi *et al.* 2012)
  - WDMs
- Operation as real-time digitizer
  - Unrestricted time window
  - Arbitrary number of independent CS measurements
  - Arbitrary RF signals
    - Optical pulses and RF signal unsynchronized
    - Pulses, chirps, sinusoids, communication waveforms ...

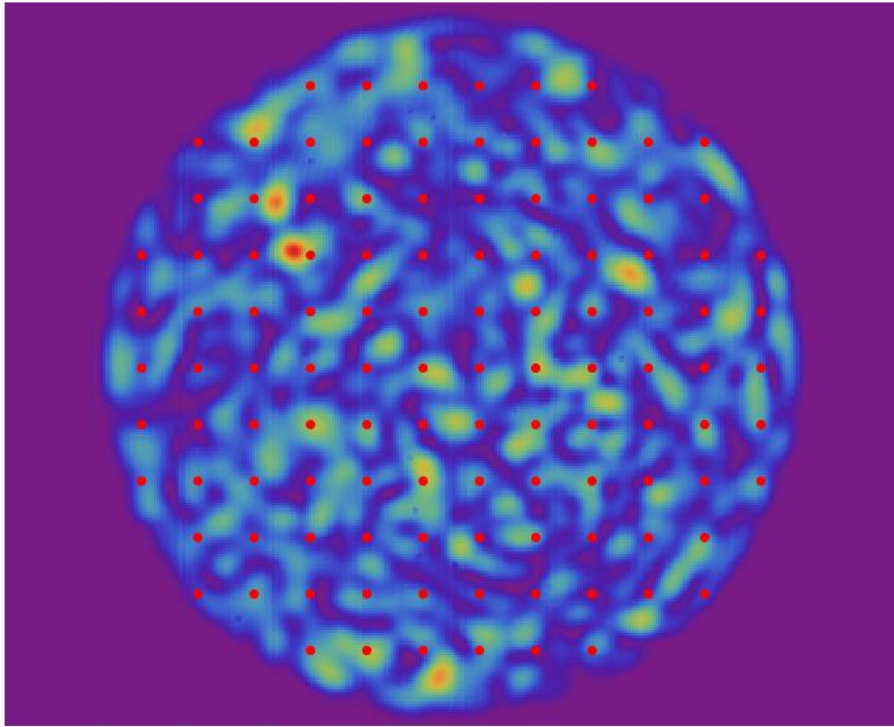
# Photonic CS system using multimode waveguide speckle



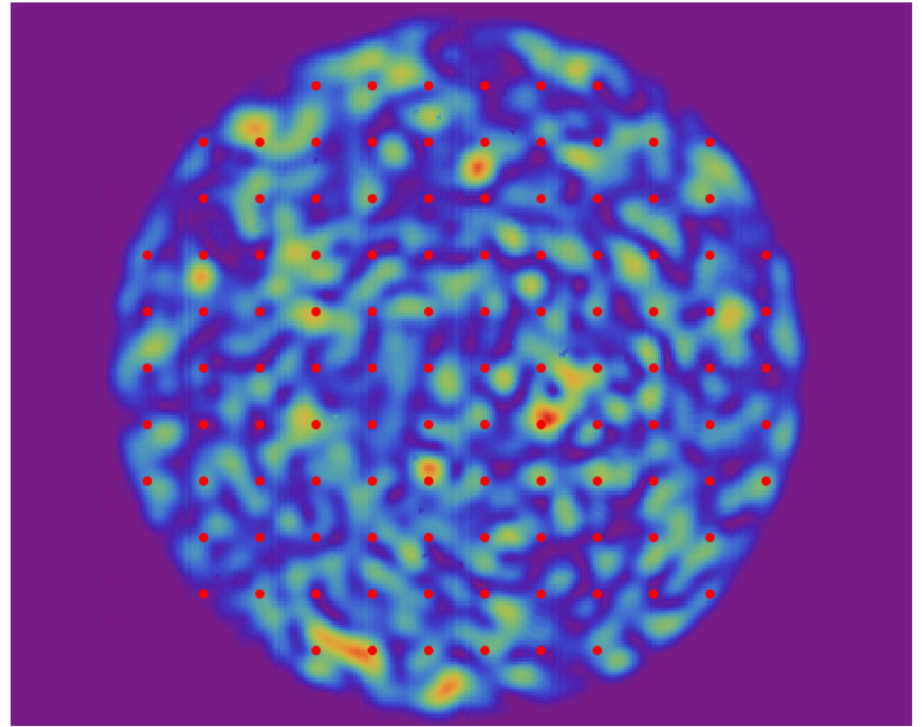
- Multimode waveguide/fiber replaces 2D SLM or WDM-Attenuator-WDMs subsystem
- Exploit spatial randomness and wavelength sensitivity at output of multimode guide
- Time-wavelength mapping multiplies RF signal by speckle wavelength dependence



# Speckle Patterns at the output of a 1m, 105 $\mu$ m, 0.22NA step-index fiber



$\lambda = 1539.44 \text{ nm}$



1539.52nm

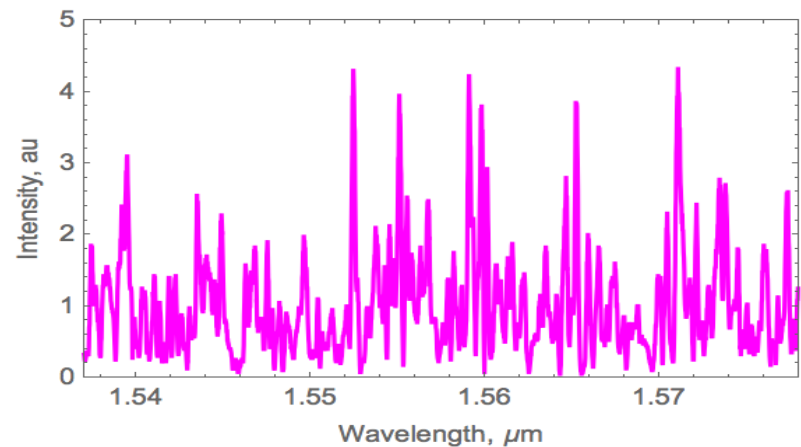
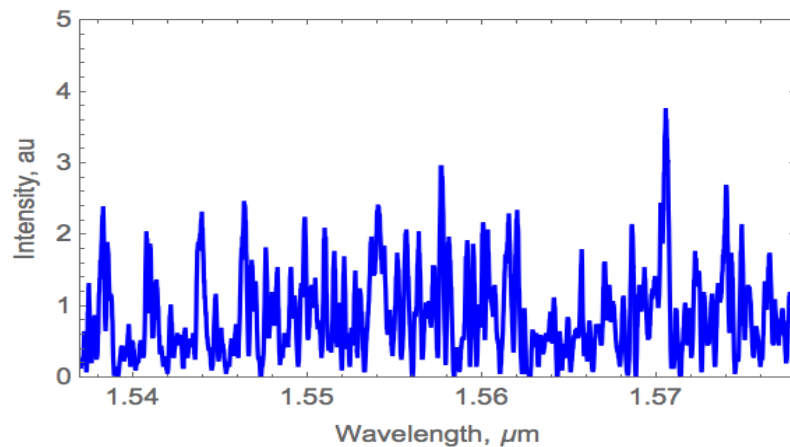
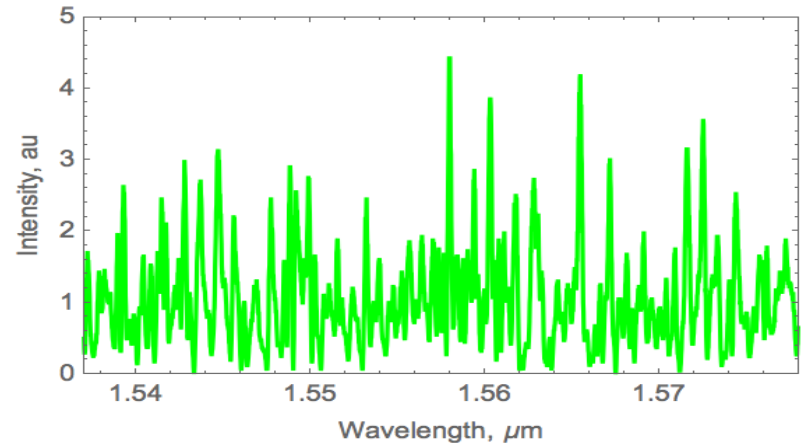
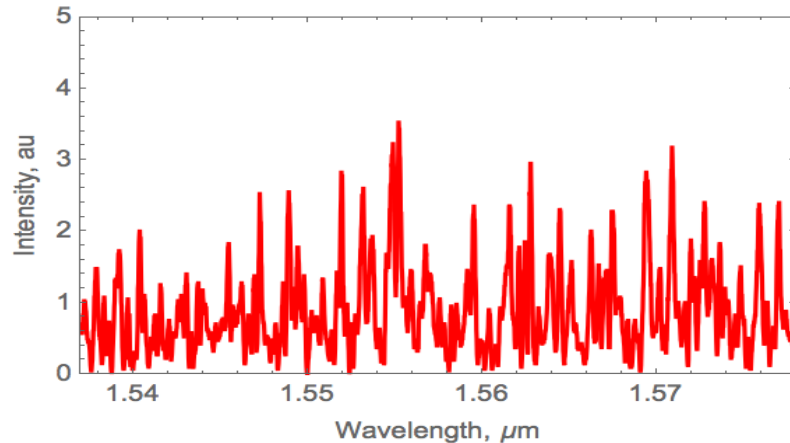
Small changes in wavelength can produce significant changes in speckle pattern

***Speckle at each red dot uncorrelated with that at other dots***



# Typical Rows in Speckle Measurement Matrix

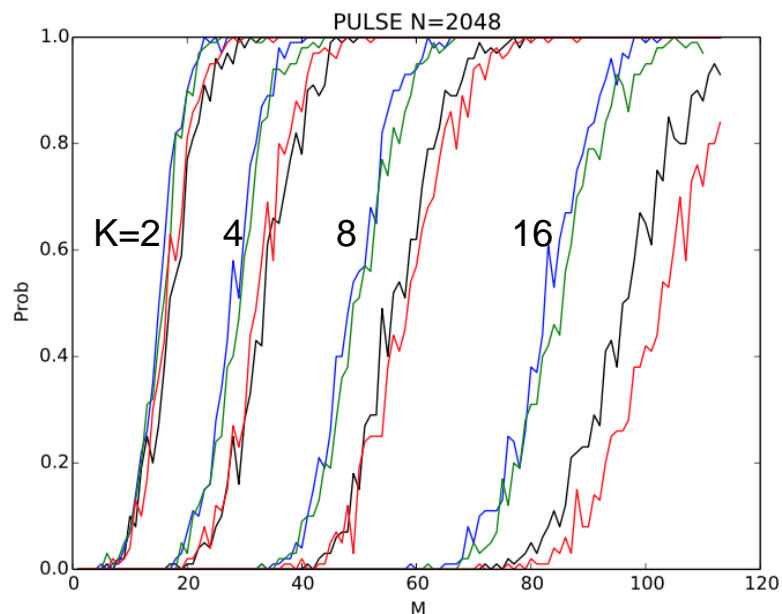
Measured intensity as a function of wavelength for 4 locations in the output plane of a 1m, 105 $\mu$ m, 0.22NA fiber



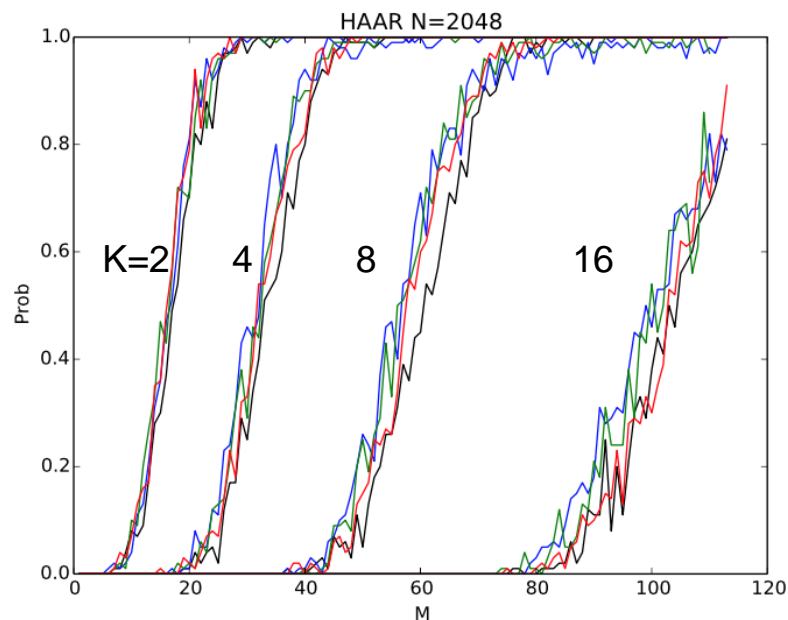
*These patterns multiply the RF signal through time-wavelength mapping*

# Simulated CS Results for Measured and Calculated Measurement Matrices (MM)

Signal sparse under Identity Transform



sparse under Haar Wavelet Transform



## Four Measurement Matrices:

- Measured for multimode fiber (1m, 105 $\mu$ m, 0.22NA)
- Calculated from Gaussian random numbers with same mean and standard deviation as measured
- Calculated MM for multimode fiber with same dimensions as measured
- Calculated MM for Planar waveguide (10cm, 25 $\mu$ m, SOI)

**Good agreement measured and calculated MMs for multimode fiber  
Speckle MMs as good as Gaussian RN MM**

# CS Probability of Recovery – 4 measurement matrices

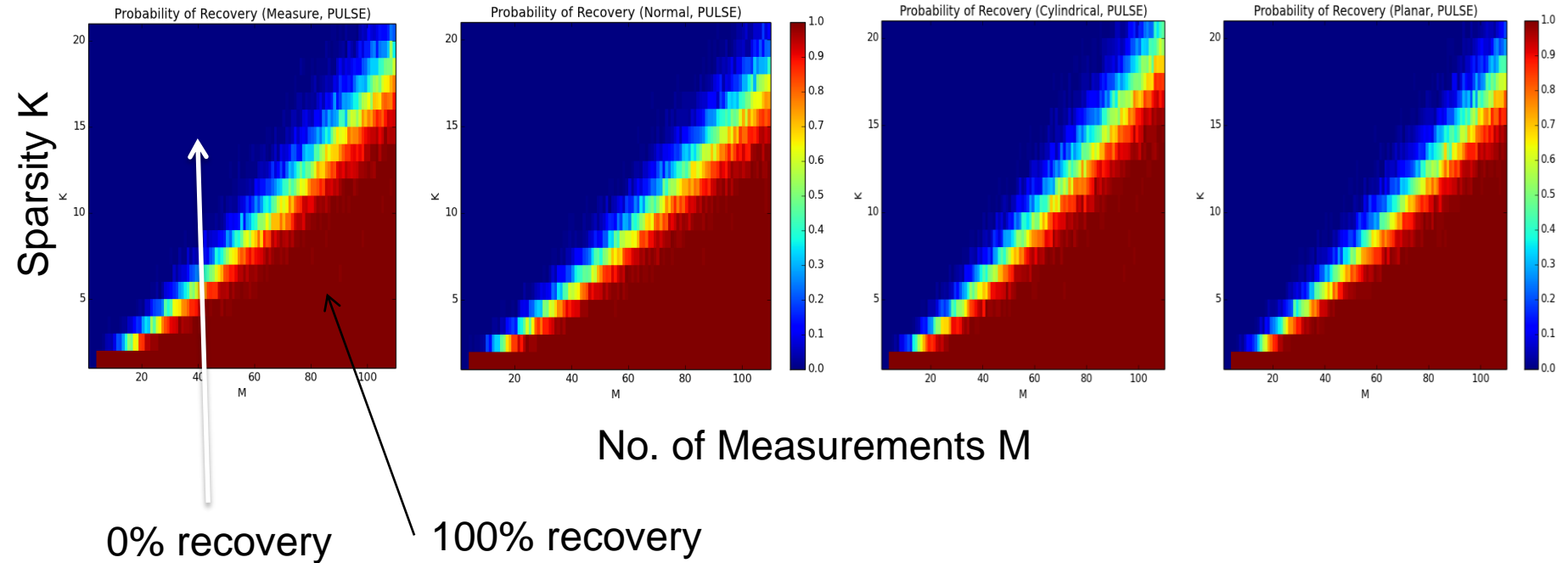
*Signals sparse under identity transform*

Measured MM  
Multimode fiber

Gaussian RN MM

Calculated MM  
Multimode Fiber

Calculated MM  
Planar Waveguide

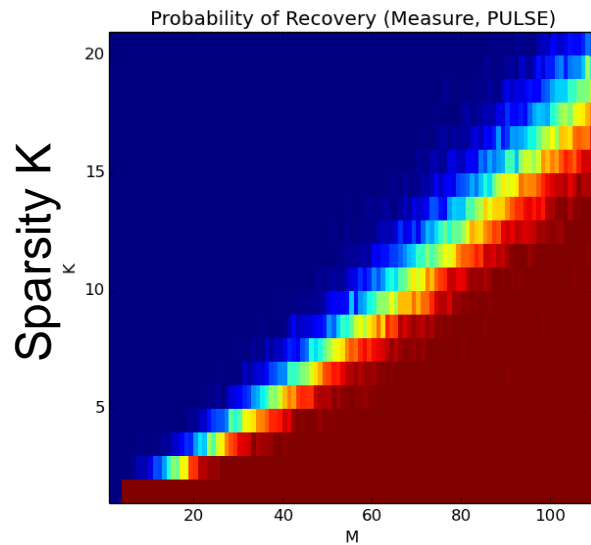


**Sharp “phase transition” from no recovery to 100% recovery typical of good CS measurement matrices**  
**Planar waveguide performance comparable to multimode fiber**

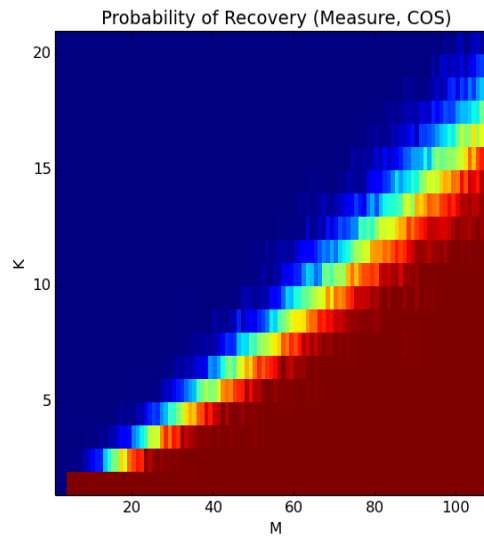
# CS Probability of Recovery – measured fiber MM

*Signals sparse under different transforms*

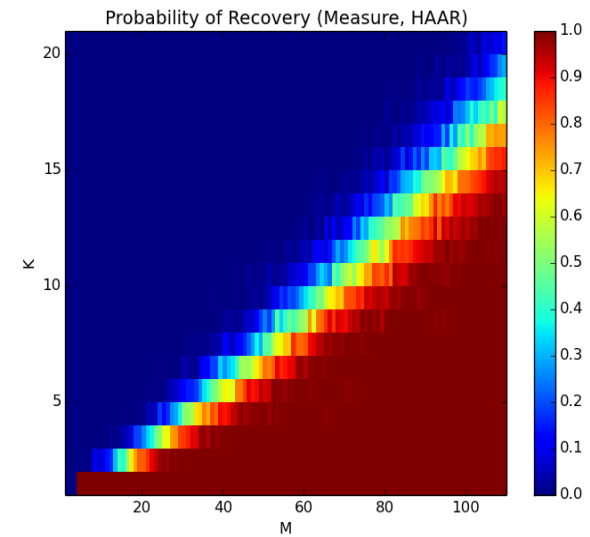
Identity



Discrete Cosine



Haar Wavelet



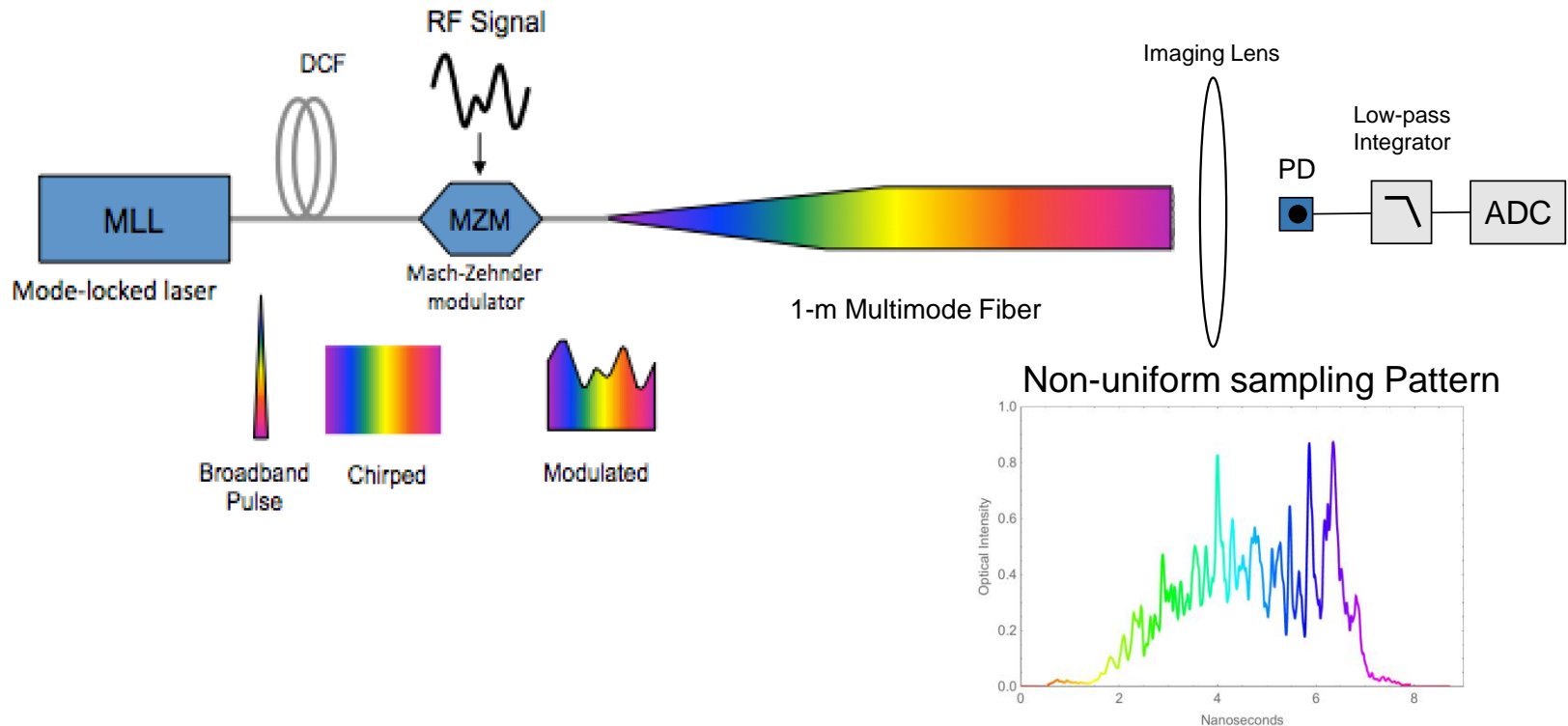
Number of Measurements  $M$

*Slightly better performance for signals comprised of pulses (Identity transform)*

# Demonstration of single channel of Speckle system

## *Periodic non-uniform Sampling with integration*

- 1-m multimode fiber
  - 105- $\mu\text{m}$  core diameter, 0.22 NA, step-index
- Single photodiode placed in image plane of fiber output
  - PD diameter at fiber output = 14  $\mu\text{m}$ , accounting for 72x image magnification

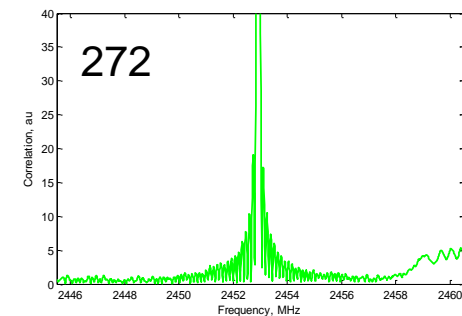
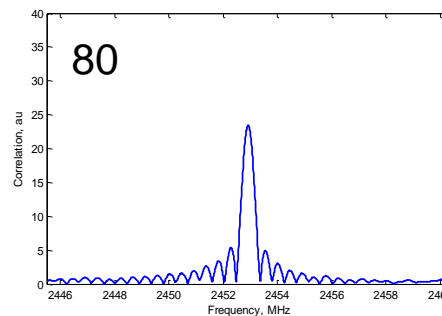
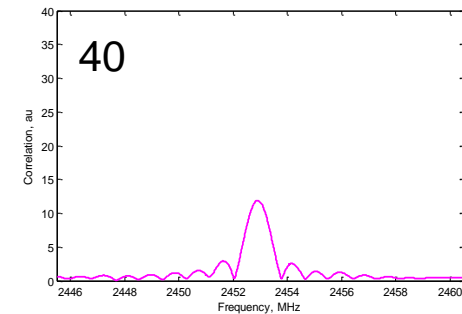
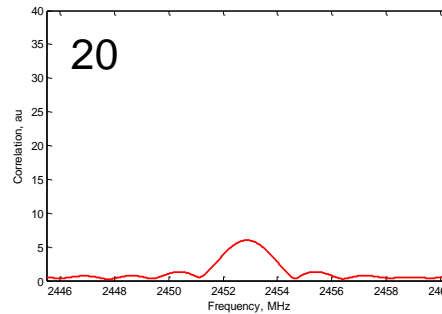
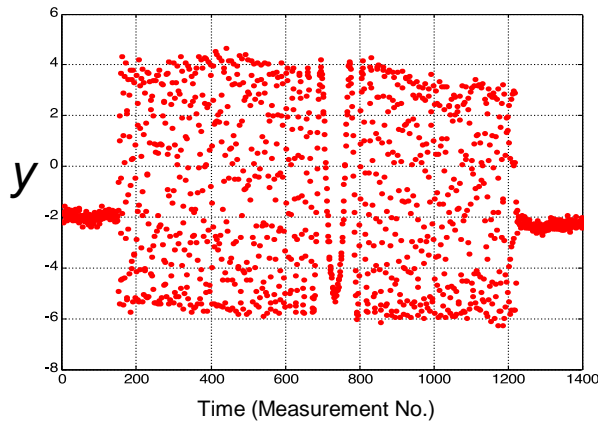


***Non-uniform sampling given by speckle pattern dependence on wavelength/time***

# Experimental results for RF Chirped pulses

- RF Parameters:
  - Chirp = 20 MHz / 30  $\mu$ s
  - Carrier Freq = 2.453 GHz
  - Maximum likelihood recovery

Carrier Recovery vs. Number of Measurements

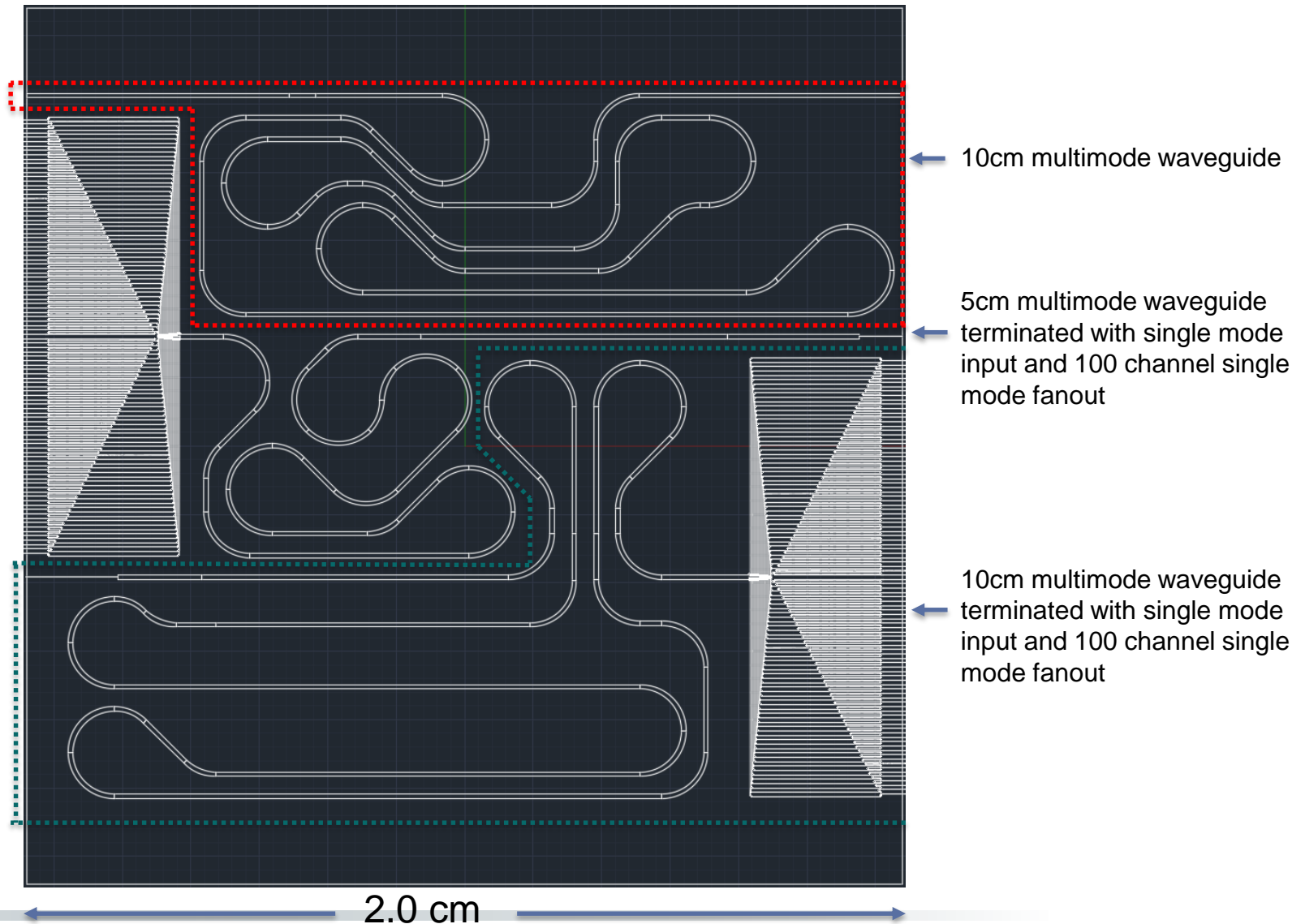


***WDM and Speckle non-uniform sampling results nearly identical***

# Next steps

- Multi-channel CS demonstration with multimode fiber
- Fabrication of Si planar waveguide
- Demonstration of CS with planar waveguide
- Integration of optical source, optical modulator, speckle waveguide and photodiodes
- Demonstration of CS with integrated system

# Compressive Sensing Receiver Reticle Design





# Other applications of laser speckle

- Machine learning
  - Saade et al. *"Random projections through multiple optical scattering: Approximating kernels at the speed of light"* arXiv 2015
- Spectroscopy
  - Redding and Cao. *"Using a multimode fiber as a high-resolution, low-loss spectrometer."* Optics letters 2012 also Optics Express 2013
- Wavelength meter
  - Mazilu et al. *"Random super-prism wavelength meter."* Optics letters 2014
- Strain sensor
  - Varyshchuk, et al. *"Using a multimode polymer optical fiber as a high sensitivity strain sensor."* 2014.
- Imaging
  - Liutkus et al. *"Imaging with nature: Compressive imaging using a multiply scattering medium."* Scientific reports 2014.
  - Kolenderska et al. *"Scanning-free imaging through a single fiber by random spatio-spectral encoding."* Optics letters 2015
  - Shin et al. *"Single-pixel imaging using compressed sensing and wavelength-dependent scattering."* Optics letters 2016.

# Conclusions

- Many undersampling strategies developed for measuring RF signals in the GHz band at sub-Nyquist rates
- Intense work in the past 8 years on photonic implementations
  - *Most multiply an RF signal by a PRBS using an EO modulator*
- Serial compressive sensing demonstrated with SLM applying PRBS
- Periodic non-uniform sampling demonstrated using
  - *WDM system*
  - *Speckle in multimode waveguide*
- Compressive sensing simulations
  - *Typical CS behavior with measured and calculated multimode fiber and calculated multimode waveguide measurement matrices*
  - *Sparsity/Masurement plots with sharp phase changes*
- Simulations for planar waveguides indicate a path to a photonic integrated circuit for a CS receiver in the GHz band