

## Some examples of problems for the Preliminary Examination in EE-Circuits & Devices EECS180

**Note that these are just some examples. Completely different problems may be given. For example note that plane wave problems can be given though there is no example given below.**

1. This exam is closed book and closed notes.
2. Show all of your work.
3. Perform all your work on the paper provided.
4. Write neatly. You will not be given credit for work that is not legible.
5. Leave answers in terms of the parameters given in the problem.
6. Circle your final answers.
7. If you have any questions, ask. You will not be given credit for work that is based on a wrong assumption.

### Problem

Consider an infinitely long lossless transmission line with  $C$  and  $L$  the capacitance and the inductance per unit length, respectively. Consider the two differential equations (the so called telegraphist equations) describing a monochromatic wave in a transmission line:

$$-\frac{dV(z)}{dz} = j\omega L I(z), \quad -\frac{dI(z)}{dz} = j\omega C V(z).$$

Derive (a) the associated wave equation; (b) the general  $V$  and  $I$  solutions; (c) the characteristic impedance and the velocity of the phase propagation. You need to prove the solutions, do not just state them. You will not receive any credit if you just write a solution without proving it.

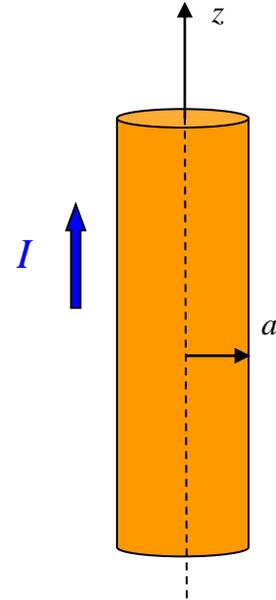
### Problem

A parallel-plate capacitor has a plate separation of  $h$  [m]. The bottom plate is at zero [V] while the top plate is at  $V_0$  [V]. Assume that  $x$  is measured vertically down from the top plate. Also assume that between the plates is a dielectric with a relative permittivity of  $\epsilon_r$ . The capacitor is assumed to have very large lateral extent (known lateral size  $a \times b$ , with  $a$  and  $b$  much larger than  $h$ ), and the plates made of perfectly electric conductor. Find (1) the electric field  $E$  and  $D$  inside (do not just write the formula, prove your result using fundamental concepts like Maxwell equations). (2) Find the surface charge density on each metal plate. (3) Derive the formula of the capacitance.

### Problem

An infinite solid wire of radius  $a$  carries a current  $I$  [A] in the  $z$  direction as shown below. Because the current is a DC current, you may assume that it is distributed uniformly inside the cross section of the wire. Assume that the wire is nonmagnetic ( $\mu = \mu_0$ ).

- Find the magnetic field vector  $\mathbf{H}$  in the region  $\rho < a$  (inside the wire) and in the region  $\rho > a$  (outside the wire).
- Find the magnetic stored energy inside the wire, over a length  $L$ .



### Problem

A solenoid of length 0.2 m and circular cross section of radius 1 cm has  $N=200$  turns of wire. The material inside the solenoid has relative permeability equal to 1000. (1) Derive the formula of the inductance using fundamental Maxwell equations, and determine the inductance value. (2) There is a time-harmonic current  $i(t) = I_0 \sin(\omega t)$  with  $I_0 = 10$  [mA] and  $\omega = 2\pi 10^5$  rad/s. What is the magnetic field inside the solenoid?

### Problem

Are the  $E$  and the  $H$  vectors (i.e., electric and magnetic fields) of a plane wave orthogonal to each other? How would you prove it?

### Problem

(i) Describe Gauss theorem in electrostatics.

(ii) A uniform charge distribution, with known volume charge density  $\rho$ , fills an infinitely long cylinder with radius  $a$ . Apply Gauss theorem to determine the electrostatic field outside (strength and direction). Discuss and show all the steps.

## Problem

- Write down the four Maxwell equations in a homogeneous isotropic medium with permittivity  $\epsilon$ , permeability  $\mu$ , and conductivity  $\sigma$ .
- Solve the Maxwell equation for the electric field of a uniform sinusoidal plane wave with radian frequency  $\omega$  propagating along the Z-axis in the medium of (a) with  $\sigma = 0$ . Determine the following physical quantities: Propagation constant, Phase velocity, Group velocity, Wavelength, Wave impedance, and Poynting vector.
- Sketch the waveform obtained in part (b) as a function of  $z$  at a fixed time  $t = t_0$  and as a function of time at a fixed plane  $z = z_0$ .
- Write down all the analogous physical quantities between a sinusoidal wave (voltage and current) propagating along a uniform lumped LC transmission line and part (b).

## Problem

Consider a uniform closely wound toroidal coil made (with circular cross section) of  $N$  turns of coil over a material with relative permeability equal to 100, and carrying a current  $I$ .

- Determine the magnetic flux density  $B$  inside.
- Determine the inductance. Assume that the radius  $R$  is much larger than  $d$ , the diameter of the circular cross section)

