Super-Orthogonal Space–Time Trellis Codes

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***Abstract*—We introduce a new class of space-time codes called super-orthogonal space-time trellis codes. These codes combine set partitioning and a super set of orthogonal space–time block codes in a systematic way to provide full diversity and improved coding gain over earlier space–time trellis code constructions. We also study the optimality of our set partitioning and provide coding gain analysis. Codes operating at different rates, up to the highest theoretically possible rate, for different number of states can be designed by using our optimal set partitioning. Super-orthogonal space-time trellis codes can provide a tradeoff between rate and coding gain. Simulation results show more than 2-dB improvements over the codes presented in the literature while providing a systematic design methodology.**

***Index Terms*—Orthogonal designs, set partitioning, space–time codes, super-orthogonal codes, transmitter diversity, trellis codes.**

I. Introduction

SPACE–TIME trellis codes have been introduced in [1] to provide improved error performance for wireless systems using multiple transmit antennas. The authors have shown that such codes can provide full diversity gain as well as additional signal-to-noise ratio (SNR) advantage that they call the coding gain. Code design rules for achieving full diversity are also provided. Using these design rules, examples of codes with full diversity as well as some coding gain were constructed that are not necessarily optimal. Since there is no general rule for designing codes that provide diversity as well as coding gain, it is unclear how to design new codes for different number of states or different rates. Also, it is not clear how to improve the performance of the codes, i.e., how to maximize the coding gain. There have been many efforts to improve the performance of the original space–time trellis codes [2]–[5]. While very interesting codes have been proposed in the literature, the coding gain improvements are marginal for one receive antenna. In this work, not only do we propose a scheme that improves the performance by more than 2 dB, but also we answer the questions of systematic design for any rate and number of states and the maximization of the coding gain. We provide a new structure for space–time trellis codes that guarantees full diversity and provides opportunity to maximize the coding gain. We also provide a systematic method to maximize the coding gain for a given rate, constellation, and number of states.

In [7], Alamouti introduced a simple code to provide full diversity for two transmit antennas. In [8], the scheme is generalized to an arbitrary number of antennas and is named space–time block coding. Also, the theory of orthogonal designs has been generalized in [8] to show when it is possible to achieve full diversity. Although a space–time block code provides full diversity and a very simple decoding scheme, despite the name, its main goal is not to provide the additional coding gain [8], [9]. This is in contrast to space–time trellis codes, which provide full diversity as well as coding gain but at a cost of higher decoding complexity. To achieve additional coding gain, one should concatenate an outer code such as a trellis code with an inner space–time block code. In [10], space–time block coding is combined with a trellis code to provide more coding gains. The same scheme is used in [11] for Rayleigh-fading channels with large space–time correlations and simulation results are provided. The shortcoming of the scheme in [10] and [11] is the fact that it does not provide the highest possible rate. The idea of concatenating a space–time block code with an outer trellis code is also exploited in [12], [13].

In what follows, we combine space–time block codes with a trellis code to come up with a new structure that guarantees the full diversity with increased rate over [10] and [11]. Also, we show how to design the trellis code to maximize the coding gain. The result is a systematic method to design space–time trellis codes for any given rate and number of states. To the best of our knowledge, this is the first systematic way of designing space–time trellis codes. Not only do we show how to design a space–time trellis code for a given rate and number of states, but also our general set-partitioning results provide the maximum coding gain for the proposed structure. During the review of this manuscript, we realized that Siwamogsatham and Fitz have independently come up with similar ideas [14]–[16]. One specific example of our general super-orthogonal space–time trellis codes is also presented in [17] independently. Part of the current work has been presented as conference papers in [18] and [19].

Section II provides the motivation behind this work. In Section III, first we provide a parameterized class of space–time block codes. Then, we study the set partitioning of space–time block codes using phase-shift keying (PSK) constellation symbols. Using the new parameterized class of space–time block codes and our set partitioning, we show how to design optimal super-orthogonal space–time trellis codes. Section IV provides an analysis of the coding gain for the super-orthogonal codes. In Section V, we extend our systematic method of designing super-orthogonal space–time trellis codes to more than two transmit antennas. We present simulation results in Section VI. Finally, some concluding remarks are provided in Section VII.

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II. Motivation

An example of a full-rate full-diversity complex space–time block code is the scheme proposed in [7] which is defined by the following transmission matrix:

$$\mathcal{C}(x_1, x_2) = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}. \tag{1}$$

The scheme can be used for $N = 2$ transmit antennas and any number of receive antennas. The scheme transmits $2b$ bits every two symbol intervals, where the two–dimensional (2-D) constellation size is $L = 2^b$. For each block, $2b$ bits arrive at the encoder and the encoder chooses two modulation symbols s_1 and s_2. Then, using $\mathcal{C}(s_1, s_2)$, the encoder transmits s_1 from antenna one and s_2 from antenna two at time one. Also, the encoder transmits $-s_2^*$ from antenna one and s_1^* from antenna two at time two. This scheme provides diversity gain, but no additional coding gain.

By concatenating an outer trellis code that has been designed for the additive white Gaussian noise (AWGN) channel with the space–time block code, additional performance gain can be obtained. To see this, view each of the 2^{2b} orthogonal matrices generated by the space–time block code (1) as a four–dimensional (4-D) signal point (strictly speaking while there are four elements in the matrix, it does not create a 4-D space and there is only two degrees of freedom). The outer trellis code's task is to select one of the 4-D signal points to be transmitted based on the current state and the $2b$ input bits. In [10], it is shown that for the slow fading channel, the trellis code should be based on the set partitioning concepts of "Ungerboeck codes" for the AWGN channel.

One shortcoming of the scheme proposed in [10] and in [11] is the fact that there is a rate loss associated with achieving any coding gain if the constituent 2-D signal constellation size does not increase. This is because these schemes are not using all of the possible 4-D signal constellations.

To elaborate, consider other codes which provide behavior similar to those of (1) for the same rate and number of transmit antennas. The set of all such codes which only use x_1, x_2, and their conjugates with positive or negative signs are listed as follows:

$$\begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}, \quad \begin{pmatrix} -x_1 & x_2 \\ x_2^* & x_1^* \end{pmatrix}, \quad \begin{pmatrix} x_1 & -x_2 \\ x_2^* & x_1^* \end{pmatrix}$$
$$\begin{pmatrix} x_1 & x_2 \\ x_2^* & -x_1^* \end{pmatrix}, \quad \begin{pmatrix} -x_1 & -x_2 \\ x_2^* & -x_1^* \end{pmatrix}, \quad \begin{pmatrix} -x_1 & x_2 \\ -x_2^* & -x_1^* \end{pmatrix}$$
$$\begin{pmatrix} x_1 & -x_2 \\ -x_2^* & -x_1^* \end{pmatrix}, \quad \begin{pmatrix} -x_1 & -x_2 \\ -x_2^* & x_1^* \end{pmatrix}. \tag{2}$$

With a small abuse of notation, we call the union of all these codes as "super-orthogonal code" set $\mathcal{C}$. Using just one of the constituent codes from $\mathcal{C}$, e.g., the code in (1), one cannot create all possible orthogonal 2×2 matrices for a given constellation. To make this point more evident, let us concentrate on binary phase-shift keying (BPSK) constellation for now. It can be shown that one can build all possible 2×2 orthogonal matrices using two of the codes in $\mathcal{C}$. For example, one can generate the following four 2×2 matrices using the code in (1):

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}. \tag{3}$$

There are four other possible distinct orthogonal 2×2 matrices which are listed as follows:

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{4}$$

To create these additional matrices, one can use the following code from the set $\mathcal{C}$:

$$\begin{pmatrix} -x_1 & x_2 \\ x_2^* & x_1^* \end{pmatrix} \tag{5}$$

which represents a phase shift of the signals transmitted from antenna one by π. We denote a set including all 2×2 orthogonal matrices from (3) and (4) as $\mathcal{O}_2$. It is important to note that the rank of a matrix (which determines diversity) based on the difference between any two distinct matrices within either (3) or (4) is 2, but the rank of a matrix obtained by considering the difference between any two elements in (3) and (4) is 1.

By using more than one code from set $\mathcal{C}$, we can create all possible 2×2 orthogonal matrices from $\mathcal{O}_2$. Therefore, the scheme provides a sufficient number of constellation matrices to design a trellis code with the highest possible rate. Also, it allows a systematic design of space–time trellis codes using the available knowledge about trellis-coded modulation (TCM) [20] and multiple TCM (MTCM) [21] in the literature.

In the space–time trellis codes of [1], N constellation symbols are assigned to each trellis branch. So, choosing a trellis branch is equivalent to transmitting N symbols from N transmit antennas in one time slot. What is transmitted at the next time slot depends on the next selected trellis branch and is not determined automatically. The codes in [1] are designed such that a maximum diversity and rate are guaranteed. However, it is not clear if the highest possible coding gain is achieved. In this work, we present new codes that not only provide maximum diversity and rate, but also achieve coding gains higher than those of the codes in [1].

In our new scheme, we assign a space–time block code with specific constellation symbols to transitions originating from a state. Therefore, in general, for a $T \times N$ space–time block code, picking a trellis branch emanating from a state is equivalent to transmitting NT symbols from N transmit antennas in T time intervals. By doing so, it is guaranteed that we get the diversity of the corresponding space–time block code, as in [10] and [11], while in what follows we show how to design the trellis code for the highest possible rate, as was done in [1], to get the maximum coding gain as well. Note that different space–time block codes can be assigned to different trellis branches as long as they all provide the same diversity (otherwise, the diversity is defined by the lowest diversity). We elaborate on the issues of picking the right space–time block codes and assigning them to different trellis branches in the following sections.

III. Super-Orthogonal Codes

A. A Parameterized Class of Space–Time Block Codes

In this work, we use the following class of orthogonal designs as transmission matrices:

$$\mathcal{C}(x_1, x_2, \theta) = \begin{pmatrix} x_1 e^{j\theta} & x_2 \\ -x_2^* e^{j\theta} & x_1^* \end{pmatrix}. \tag{6}$$

Note that $\theta = 0$ provides the code in (1). So, with a slight abuse of notation, we have $\mathcal{C}(x_1, x_2, 0) = \mathcal{C}(x_1, x_2)$. Let us concentrate on the case for which the constellation signal alphabet is not expanded. Note that the transmitted signals for $\mathcal{C}(x_1, x_2, \theta)$ are $x_1 e^{j\theta}$, x_2, $-x_2^* e^{j\theta}$, and x_1^*. We pick θ such that for any choice of x_1 and x_2 from the original constellation points, the resulting *transmitted* signals are also from the same constellation. For example, if we use L-PSK constellation signals (PSK constellations containing L points), the constellation signals (and thus x_1 and x_2) can be represented by $e^{\frac{j2\pi l}{L}}$, $l = 0, 1, \ldots, L-1$. One can pick $\theta = 2\pi l'/L$, where $l' = 0, 1, \ldots, L-1$. In this case, the resulting transmitted signals are also members of the L-PSK constellation and, therefore, do not expand the constellation signals. Since the transmitted signals are from a PSK constellation, the peak-to-average power ratio of the transmitted signals is equal to one. So, not only do we not increase the number of signals in the constellation, but also there is no need for an amplifier to provide a higher linear operation region. More specifically, we use $\theta = 0, \pi$ and $\theta = 0, \pi/2, \pi, 3\pi/2$ for BPSK and quaternary PSK (QPSK), respectively. By using $\mathcal{C}(x_1, x_2, 0)$ and $\mathcal{C}(x_1, x_2, \pi)$ for the BPSK constellation, one can generate all 2×2 orthogonal matrices in $\mathcal{O}_2$. In fact, $\mathcal{C}(x_1, x_2, 0)$ is the code in (1) and $\mathcal{C}(x_1, x_2, \pi)$ is the code in (5). By using (1) and (5), the set of transmitted signals consists of $x_1, x_2, x_1^*, x_2^*, -x_1, -x_2^*$. For any symmetric constellation, the set of transmitted signals is the same as the set of constellation signals. This includes quadrature amplitude modulation (QAM) constellations as well as PSK constellations. We call the combination of these two codes a super-orthogonal code. In general, a super-orthogonal code consists of the union of a few orthogonal codes, like the ones in (6). A special case is when the super-orthogonal code consists of only one orthogonal code, e.g., only $\theta = 0$. Therefore, the set of orthogonal codes is a subset of the set of super-orthogonal codes. Obviously, the number of orthogonal matrices that a super-orthogonal code provides is more than (or in the worst case equal to) the number of orthogonal matrices that an orthogonal code provides. Therefore, while the super-orthogonal code does not extend the constellation alphabet of the transmitted signals, it does expand the number of available orthogonal matrices. This is of great benefit and crucial in the design of full-rate, full-diversity trellis codes. Another advantage of super-orthogonal codes lies in the fact that the code is parameterized. An important example of a super-orthogonal code that we use throughout this paper is the union of $\mathcal{C}(x_1, x_2, \theta)$ for an L-PSK constellation where $\theta = 2\pi l'/L$, and $l' = 0, 1, \ldots, L-1$. Another example is the union of $\mathcal{C}(x_1, x_2, \theta)$ for a QAM constellation where $\theta = \pi/2, \pi, 3\pi/2$.

Note that one needs to separate "the expansion of the orthogonal matrices" that is a positive result of using super-orthogonal codes and "the expansion of the signal constellation" that is usually a negative side effect. Our super-orthogonal code expands the number of available orthogonal matrices and, as we show in the sequel, this is the main reason why we can design full-rate trellis codes that provide full diversity. It has no negative side effect either in terms of expanding the transmitted signal constellation or in terms of increasing the peak-to-average power ratio.

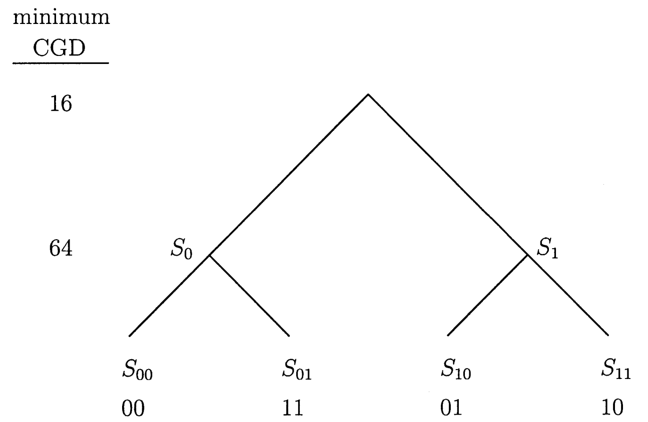

Fig. 1. Set partitioning for BPSK; the numbers at leaves represent the indexes of the symbols in the space–time block code.

B. Set Partitioning for Orthogonal Codes

This section provides a set partitioning for orthogonal codes and shows how to maximize the coding gain. Let us denote the difference of the transmission matrices for codewords c_1 and c_2 by $B(c_1, c_2)$ and its Hermetian, complex conjugate and transpose, by $B^H(c_1, c_2)$. Following the definitions in [1], the diversity of such a code is defined by the minimum rank of the matrix $B(c_1, c_2)$. For a full-diversity code, the minimum of the determinant of the matrix $A(c_1, c_2) = B(c_1, c_2)B^H(c_1, c_2)$ over all possible pairs of distinct codewords c_1 and c_2 corresponds to the coding gain. We define the coding gain distance (CGD) between codewords c_1 and c_2 as $d^2(c_1, c_2) = \det(A(c_1, c_2))$, where $\det(A)$ is the determinant of matrix A. In general, if instead of a full diversity N we have a code with diversity less than N, the distance can be defined as the harmonic mean of the nonzero eigenvalues of $A(c_1, c_2)$. Then, we use CGD instead of Euclidean distance to define a set partitioning similar to Ungerboeck's set partitioning [20].

Let us concentrate on the case where we only utilize the code in (1). Also, let us assume that we use a BPSK constellation. Consider a four-way partitioning of the orthogonal code as shown in Fig. 1 for BPSK. At the root of the tree, the minimum determinant is 16. At the first level of partitioning, the highest determinant that can be obtained is 64. This is obtained by a set partitioning in which subsets S_0 and S_1 use different transmitted signal elements for different transmit antennas. At the next level of partitioning, we have four sets S_{00}, S_{11}, S_{01}, and S_{10} with only one element per set.

A four-state trellis code (Fig. 2) can be constructed based on the set partitioning of Fig. 1 with a rate of $b/2$ bits per second per hertz (bits/s/Hz) for an L-PSK constellation where $L = 2^b$. This is similar to the codes presented in [10] and [11] which can

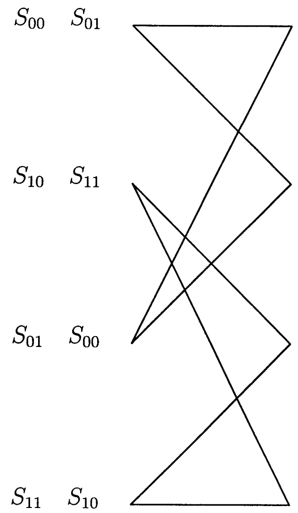

Fig. 2. A four-state trellis; Ungerboeck's set partitioning.

only transmit a rate which is half of the maximum possible rate [1]. For an L-PSK constellation, each signal can be represented by $s = e^{\frac{j2\pi l}{L}}$, $l = 0, 1, \ldots, L$ or $s = e^{jl\omega}$, where $\omega = \frac{2\pi}{L}$. Now, we consider two distinct pairs of constellation symbols

$$(s_1^1 = e^{jk_1\omega}, s_2^1 = e^{jl_1\omega}) \quad \text{and} \quad (s_1^2 = e^{jk_2\omega}, s_2^2 = e^{jl_2\omega})$$

and the corresponding code matrices c_1 and c_2 to calculate $B(c_1, c_2)$ and $A(c_1, c_2)$. For the sake of brevity, in the sequel, we omit (c_1, c_2) from A and B when there is no ambiguity. For parallel transitions in a trellis, we have (7) and (8) as shown at the bottom of the page. Using (8), one can show that

$$\det(A) = \{4 - 2\cos[\omega(k_2 - k_1)] - 2\cos[\omega(l_2 - l_1)]\}^2. \quad (9)$$

Now, if we have two codewords which differ in P pairs of constellation symbols, it can be shown that still $A_{12} = A_{21} = 0$. Also, if for the first codeword, we denote the set of constellation symbols by

$$(s_1^1, s_2^1)^p = (e^{jk_1^p\omega}, e^{jl_1^p\omega}), \qquad p = 1, 2, \ldots, P$$

and for the second codeword, we denote the set of constellation symbols by

$$(s_1^2, s_2^2)^p = (e^{jk_2^p\omega}, e^{jl_2^p\omega}), \qquad p = 1, 2, \ldots, P$$

we have

$$\det(A) = \left\{\sum_{p=1}^{P} 4 - 2\cos[\omega(k_2^p - k_1^p)] - 2\cos[\omega(l_2^p - l_1^p)]\right\}^2. \quad (10)$$

Note that (10) includes a sum of P terms and each of these terms is nonnegative. Therefore, the following inequality holds:

$$\det(A) = \left\{\sum_{p=1}^{P} 4 - 2\cos[\omega(k_2^p - k_1^p)] - 2\cos[\omega(l_2^p - l_1^p)]\right\}^2 \geq \sum_{p=1}^{P} \{4 - 2\cos[\omega(k_2^p - k_1^p)] - 2\cos[\omega(l_2^p - l_1^p)]\}^2. \quad (11)$$

Based on the coding distances calculated in (9) and (10), one can show that the coding gain of such a space–time trellis code is dominated by parallel transitions. The optimal set partitioning for BPSK, QPSK, and 8-PSK are demonstrated in Figs. 1, 3, and 4, respectively. As can be seen from these figures, the minimum CGD increases (or remains the same) as we go one level down in the tree. The branches at each level can be used to design a trellis code with a specific rate. Higher coding gain necessitates the use of redundancy resulting in reduced rate. In the following sections, we show how to design space–time trellis codes without sacrificing the rate.

C. Set Partitioning for Super-Orthogonal Codes

This subsection provides a set partitioning for super-orthogonal codes and shows how to maximize the coding gain without sacrificing the rate. Code construction based on a super-orthogonal set is as follows. In our new scheme, we assign a constituent space–time block code to all transitions from a state. The adjacent states are typically assigned to one of the other constituent space–time block codes from the super-orthogonal code. Similarly, we can assign the same space–time block code to branches that are merging into a state. It is thus assured that any path that diverges from (or merges to) the correct path differs by rank 2. In other words, every pair of codewords diverging from (or merging to) a state achieves full diversity because the pair is from the same orthogonal code (same parameter θ). On the other hand, for codewords with different θ, it is possible that they do not achieve full diversity. Since these codewords are assigned to different states, the resulting trellis code would provide full diversity despite the fact that a pair of codewords in a super-orthogonal code may not achieve full diversity. Note that this is just a general method to guarantee full diversity. It is possible to come up with examples that do not follow this rule and still provide full diversity.

Similar to the case of orthogonal designs, it remains to do the set partitioning such that the CGD is maximized at each level of partitioning. We use the formulas that we have developed

$$B = \begin{pmatrix} e^{jk_1\omega} - e^{jk_2\omega} & e^{jl_1\omega} - e^{jl_2\omega} \\ e^{-jl_2\omega} - e^{-jl_1\omega} & e^{-jk_1\omega} - e^{-jk_2\omega} \end{pmatrix} \quad (7)$$

$$A = \begin{pmatrix} 4 - 2\cos[\omega(k_2 - k_1)] - 2\cos[\omega(l_2 - l_1)] & 0 \\ 0 & 4 - 2\cos[\omega(k_2 - k_1)] - 2\cos[\omega(l_2 - l_1)] \end{pmatrix}. \quad (8)$$

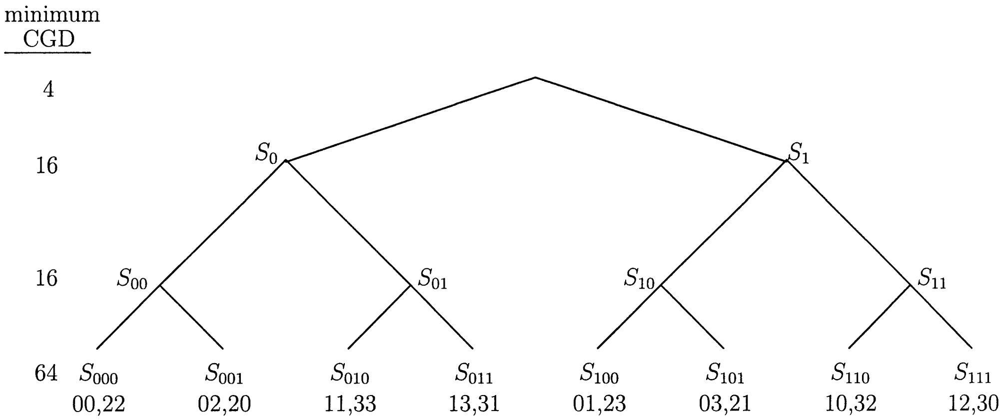

Fig. 3. Set partitioning for QPSK; the numbers at leaves represent the indexes of the symbols in the space–time block code.

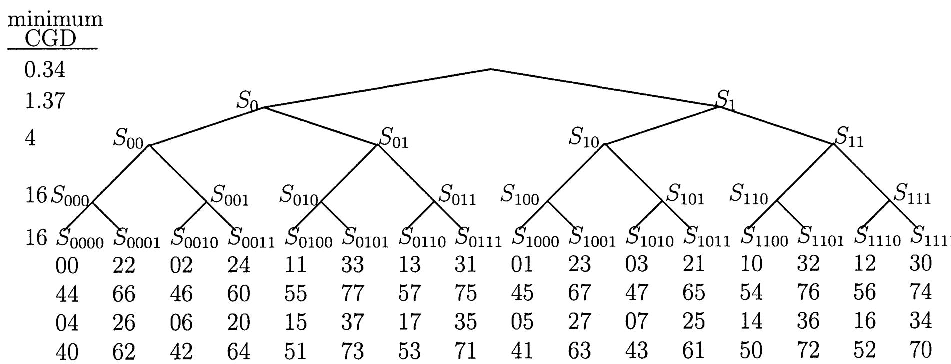

Fig. 4. Set partitioning for 8-PSK; the numbers at leaves represent the indexes of the symbols in the space–time block code.

in Section III-B to calculate the CGDs. Figs. 1, 3, and 4 show the set partitioning for BPSK, QPSK, and 8-PSK, respectively. Using (9) to calculate the CGD between a pair of codewords, it is apparent that increasing the Euclidean distance between the first signals of the codewords will increase the CGD. The CGD also increases as we increase the Euclidean distance between the second signals of the codewords. Therefore, a rule of thumb in set partitioning is to choose the codewords that contain signal elements with highest maximum Euclidean distance from each other as the leaves of the set-partitioning tree. For example, in the case of QPSK in Fig. 3, $s = e^{\frac{j\pi l}{2}}$, $l = 0, 1, 2, 3$ are the QPSK signal constellation elements and $k = 0, 1, 2, 3$ represent $s = 1, j, -1, -j$, respectively. The maximum CGD in this case is 64 when $|k_1 - k_2| = 2$ and $|l_1 - l_2| = 2$ in (9). This is the justification for the choice of leaf codewords in Fig. 3. At the second level of the tree from the bottom, it is impossible to have both $|k_1 - k_2|$ and $|l_1 - l_2|$ equal to 2 in all cases. The next highest value for CGD is 16 when $|k_1 - k_2| = 2$, $|l_1 - l_2| = 0$ or $|k_1 - k_2| = 0$, $|l_1 - l_2| = 2$. Therefore, we group the subtrees in the second level such that the worst case is when $|k_1 - k_2| = 2$, $|l_1 - l_2| = 0$ or $|k_1 - k_2| = 0$, $|l_1 - l_2| = 2$. We keep grouping the subtrees to maximize the minimum CGD at each level of set partitioning. Similar strategies are used for other signal constellations.

In Figs. 1, 3, and 4, set partitioning is used to assign signal elements to branches diverging from (or merging to) a state to maximize coding gain. The optimality of these set partitioning is discussed in Section IV.

D. Super-Orthogonal Space–Time Trellis Codes

In this section, we show how to use our proposed set partitioning scheme to design full-diversity full-rate space–time trellis codes. First, we start with a few important examples and then we propose some general rules how to design a super-orthogonal space–time trellis code for a given trellis and required rate. Figs. 5–10 demonstrate examples of our new super-orthogonal space–time trellis codes. In these figures, $\mathcal{C}(x_1, x_2, \theta)$ represents the particular member of our parameterized space–time block code which is used at the specific state. Also, we have shown the corresponding sets from our set partitioning next to each state.

Fig. 5 shows a four-state example of our new super-orthogonal space–time trellis codes. In this example, when we use BPSK and the corresponding set partitioning in Fig. 1, the rate of the code is one. We use $\mathcal{C}(x_1, x_2, 0)$ when departing from states zero and two and use $\mathcal{C}(x_1, x_2, \pi)$ when departing from states one and three. Note that, with this new structure, we have eight possible orthogonal 2×2 matrices instead of four which allows us to design a full-rate code. The minimum CGD of this code is 64 which can be found in Fig. 1 and Table I. In Section IV, we prove that parallel transitions are dominant in calculating the minimum CGD for this code.

If we use a QPSK constellation and the corresponding set partitioning in Fig. 5, the result is a four-state super-orthogonal space–time trellis code at rate 2 bits/s/Hz. The minimum CGD for this 2-bits/s/Hz code is equal to 16 which is greater than 4, the CGD of the corresponding space–time trellis code from [1].

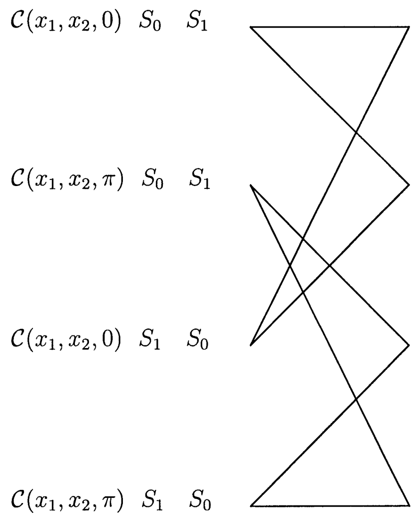

Fig. 5. A four-state code; $r = 1$ bit/s/Hz using BPSK or $r = 2$ bits/s/Hz using QPSK.

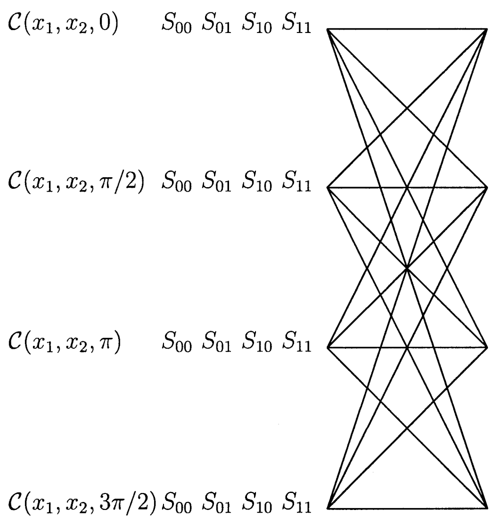

Fig. 6. A four-state code; $r = 3$ bits/s/Hz (8-PSK).

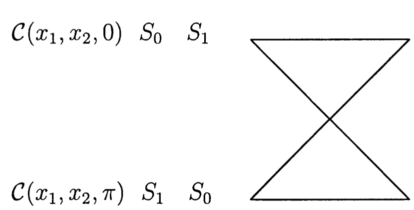

Fig. 7. A two-state code; $r = 1$ bit/s/Hz using BPSK or $r = 2$ bits/s/Hz using QPSK.

Fig. 6 shows a 3-bits/s/Hz super-orthogonal space–time trellis code using 8-PSK and the corresponding set partitioning of Fig. 4. The minimum CGD for this code is equal to 2.69. We study the details of the coding gain calculations in Section IV. There is no four-state space–time trellis code for 8-PSK in [1].

Fig. 7 demonstrates the codes for a two-state trellis providing a minimum CGD of 48 and 16 at rates 1 bit/s/Hz using BPSK and 2 bits/s/Hz using QPSK, respectively. There are no equivalent codes reported in [1].

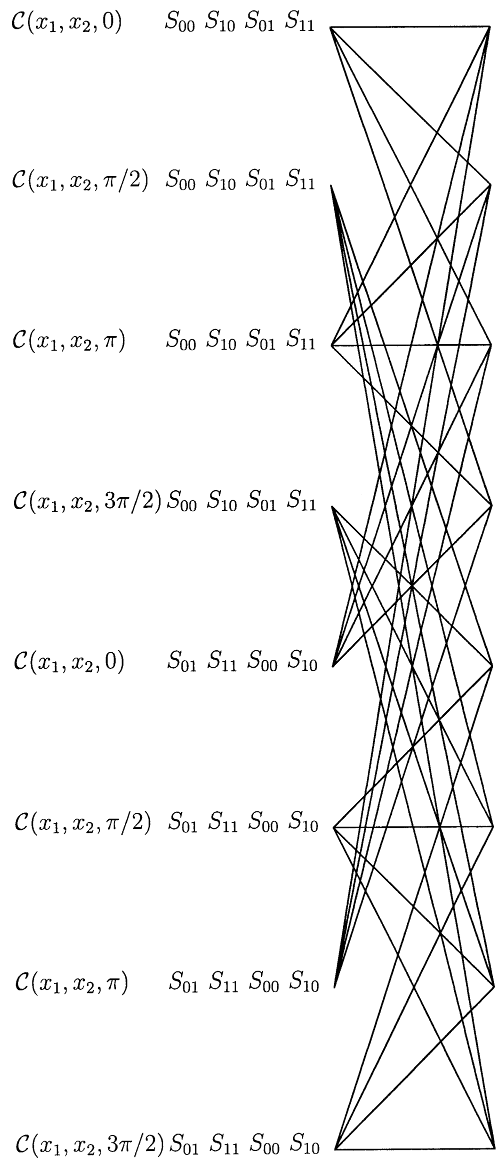

Fig. 8. An eight-state code; $r = 3$ bits/s/Hz (8-PSK).

We also provide an eight-state, rate 3-bits/s/Hz (8-PSK) example in Fig. 8. The CGD of this code is 4 while the CGD of a similar code in [1] is 2. To design such a code, we have some degrees of freedom in choosing the rotations and the sets. Therefore, there are different code designs that provide the same coding gain. Another example of an eight-state, rate 3-bits/s/Hz code with similar properties is provided in Fig. 9. Note that the only limitation in picking different options is to avoid a catastrophic code. To avoid a catastrophic code, a change of a few input bits should not create an infinite number of different symbols. In other words, the same input bits should not create the same codeword when starting from different states. To achieve this goal, either the θ for the orthogonal code assigned to different states should be different or the assigned subsets should be different. As the number of states grows, we can pick between these two options (or their combinations) and this is basically the main difference between Figs. 8 and 9. This is a good example to show the possibility of having different assignments in the code. In fact, there are choices of rotations that

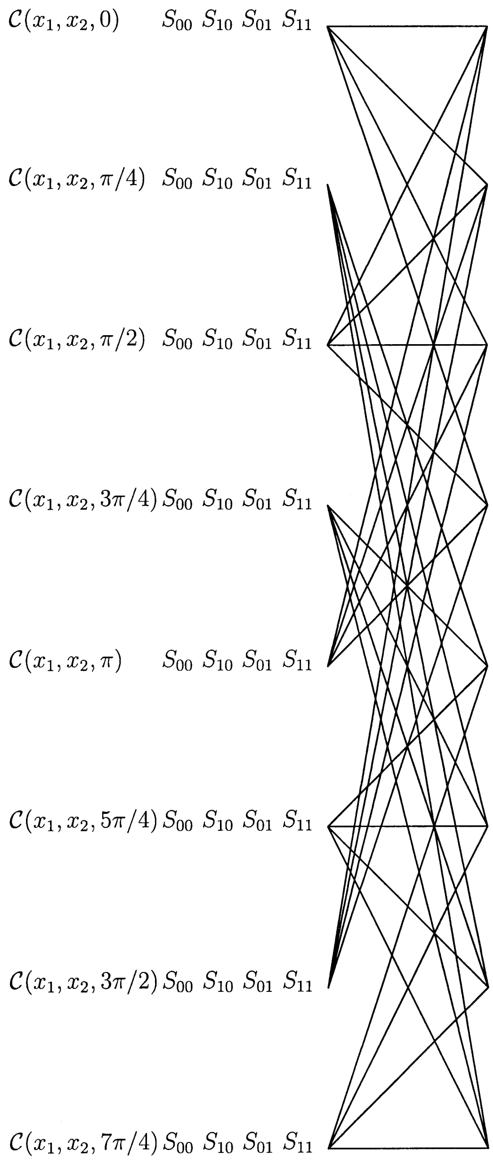

Fig. 9. An eight-state code; $r = 3$ bits/s/Hz (8-PSK).

make the overall CGD less than 4 and must be avoided. If a higher coding gain is required at $r = 3$ bits/s/Hz, one should use a 16-state super-orthogonal space–time trellis code. Similar to conventional trellis codes, there is always a tradeoff between coding gain and the number of states (complexity).

Codes with different number of states and at different rates can be systematically designed using the set partitioning in Figs. 1, 3, and 4. The rules for assigning different sets to different transitions in the trellis are similar to the general rules of thumb defined in [20] and [21] to design MTCM schemes. First, based on the required rate, we select a constellation and use the corresponding set partitioning. The choice of the constellation also defines the valid rotation angles θ that would not create an expansion of the transmitted signal constellation. We use these valid values for θ in (6) to define the super-orthogonal code to be used in the trellis code. Then, we assign a constituent orthogonal code from the set of super-orthogonal codes to all transitions from a state. The adjacent states are typically assigned to another constituent orthogonal code from the super-orthogonal code. Similarly, we can assign the same orthogonal code to branches that are merging into a state. It is thus assured that any path that diverges from (or merges to) the correct path is full rank. Then different sets from the corresponding set partitioning are assigned to different transitions similar to the way that we assign sets in a regular MTCM. To avoid a catastrophic code, the subsets are assigned such that for the same input bits in different states either the rotation angles (θ) of the orthogonal codes are different or the assigned subsets are different. Since the process of set partitioning is done to maximize the CGD at each level, the coding gain of the resulting super-orthogonal space–time trellis code is also maximum.

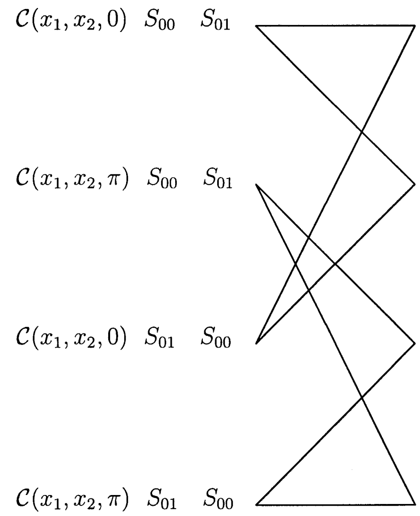

Fig. 10. A four-state code; $r = 2.5$ bits/s/Hz (8-PSK).

Note that unlike the manual design strategy in [1], one can systematically design a code for an arbitrary trellis and rate using our set partitioning and code design strategy. While we have shown the examples of full-rate codes, in general, codes with lower rates can be designed to provide higher coding gains. The design method is exactly the same. We only use sets from a different level of the set partitioning. Using different levels of the set partitioning to design super-orthogonal space–time trellis codes provides a tradeoff between rate and coding gain. For example, a four-state rate 2.5 bits/s/Hz code using 8-PSK with a CGD of 4 is shown in Fig. 10. The maximum possible rate using 8-PSK is 3 and an example is shown in Fig. 8.

Table I tabulates the CGD of our new super-orthogonal space–time trellis codes and the corresponding codes from [1]. It is evident from Table I that using our super-orthogonal space–time trellis coding method, not only can we systematically design codes that did not exist before, but also the CGD of the new codes are consistently better than those of the original codes presented in [1]. In Section VI, we compare the performance of our super-orthogonal space–time trellis codes with the best available space–time trellis codes in the literature.

IV. CGD Analysis

In this section, we derive the coding gain of different super-orthogonal space–time trellis codes that we introduced in Section III. We show how to find the dominant path for CGD calcu-

TABLE I
CGD VALUES FOR DIFFERENT CODES

Figure	No. of states	rate (bits/sec/Hz)	minimum CGD	minimum CGD in [1]
5 (BPSK)	4	1	64	-
5 (QPSK)	4	2	16	4
6	4	3	2.69	-
7 (BPSK)	2	1	48	-
7 (QPSK)	2	2	16	-
8,9	8	3	4	2
10	4	2.5	4	-

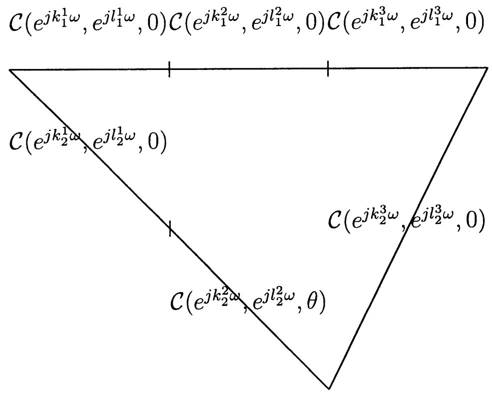

Fig. 11. Two typical paths differing in $P = 3$ transitions.

lation in the trellis. Our approach is general enough to be easily extended to other trellises in the literature. We first consider specific examples given in Section III and calculate their coding gains. Then, we generalize the methods that we have used in the calculation of these coding gains to show how to calculate the coding gain of any super-orthogonal space–time trellis code.

A. Error Events With Path Length of Three

Let us first consider the trellis in Fig. 2 which corresponds to the code in Fig. 5. Note that when there are parallel transitions between two states, we assign a different transition to each possible constellation matrix (symbols) which is defined by our set partitioning. Two codewords may only differ in $P = 1$ trellis transition. However, due to the structure of the trellis, it is impossible to have two codewords which differ in two trellis transitions $(P = 2)$. Because, for example, if two codewords diverge from state zero, they have to go through at least three transitions to re-emerge (Fig. 11). Therefore, the smallest value of P excluding parallel transitions is three. For $P = 3$, we consider a typical case where the first codeword stays at state zero. For the second codeword, the first and third transitions (diverging and merging to state zero) use $C(x_1, x_2, 0)$ and the second transition uses $C(x_1, x_2, \theta)$ as in Fig. 11. Using trigonometry equations, it can be shown that

$$\det(A) = (a + b_1 + c)(a + b_2 + c) - d \tag{12}$$

where

$$a = 4 - 2\cos\left[\omega(k_2^1 - k_1^1)\right] - 2\cos\left[\omega(l_2^1 - l_1^1)\right]$$
$$c = 4 - 2\cos\left[\omega(k_2^3 - k_1^3)\right] - 2\cos\left[\omega(l_2^3 - l_1^3)\right]$$
$$b_1 = 4 - 2\cos\left[\omega(k_2^2 - k_1^2) + \theta\right] - 2\cos\left[\omega(l_2^2 - l_1^2)\right]$$
$$b_2 = 4 - 2\cos\left[\omega(k_2^2 - k_1^2)\right] - 2\cos\left[\omega(l_2^2 - l_1^2) - \theta\right]$$
$$d = (2 - 2\cos\theta)\left(2 - 2\cos\left[\omega(k_2^2 - k_1^2 + l_1^2 - l_2^2) + \theta\right]\right). \tag{13}$$

For $\theta = \pi$, for example, the trellis in Fig. 5, we have

$$b_1 = 4 + 2\cos\left[\omega(k_2^2 - k_1^2)\right] - 2\cos\left[\omega(l_2^2 - l_1^2)\right]$$
$$b_2 = 4 - 2\cos\left[\omega(k_2^2 - k_1^2)\right] + 2\cos\left[\omega(l_2^2 - l_1^2)\right]$$
$$d = 8\left(1 + \cos\left[\omega(k_2^2 - k_1^2 + l_1^2 - l_2^2)\right]\right). \tag{14}$$

Since $a, b_1, b_2, c, d \geq 0$, we have

$$\min\det(A) \geq (\min a + \min b_1 + \min c) \cdot (\min a + \min b_2 + \min c) - \max d. \tag{15}$$

Now, we prove the following result that is used in the calculation of the coding gains.

For the four-state code in Fig. 5, the minimum value of the CGD when $P = 3$ is greater than the minimum value of the CGD when $P = 1$.

Proof: Without loss of generality, we assume two codewords diverging from state zero and re-emerging after P transitions to state zero. For parallel transitions $(P = 1)$, one can calculate the CGD from (9). In this case, $\min\det(A) = 64$ for the $r = 1$-bit/s/Hz code using BPSK and $\min\det(A) = 16$ for the $r = 2$-bits/s/Hz code using QPSK. For the $r = 1$-bit/s/Hz code using BPSK, $\omega = \pi$ and we have $\min a = \min c = 4$, $\min b_1 = \min b_2 = 4$, and $\max d = 16$. Therefore, inequality (15) results in

$$\min\det(A) \geq 128. \tag{16}$$

Also, for

$$k_2^1 = k_1^1 = l_1^1 = k_2^2 = k_1^2 = l_1^2 = l_2^2 = k_2^3 = k_1^3 = l_1^3 = 0$$

and $l_2^1 = l_2^3 = 1$ in (12), we have $\det(A) = 128$ which means

$$\min(A) \leq 128. \tag{17}$$

Combining inequalities (16) and (17) provides

$$\min(A) = 128 \tag{18}$$

which is greater than 64.

For the $r = 2$-bits/s/Hz code using QPSK, $\omega = \pi/2$ and we have $\min a = \min c = 2$, $\min b_1 = \min b_2 = 4$, and $\max d = 16$. Therefore, inequality (15) results in

$$\min\det(A) \geq 48. \tag{19}$$

Also, for

$$k_2^1 = k_1^1 = l_1^1 = k_2^2 = k_1^2 = l_1^2 = l_2^2 = k_2^3 = k_1^3 = l_1^3 = 0$$

TABLE II
MINIMUM $\det(A)$ FROM (24) FOR DIFFERENT CONSTELLATIONS AND ROTATIONS

constellation	θ	S_1^1, S_1^2	S_2^1	S_2^2	min det(A)	parallel CGD
BPSK	π	S_0	S_1	S_1	48	64
BPSK	π	S_0	S_1	S_0	48	64
QPSK	π	S_0	S_1	S_1	24	16
QPSK	π	S_0	S_1	S_0	20	16
QPSK	$\pi/2, 3\pi/2$	S_0	S_1	S_1	12	16
QPSK	$\pi/2, 3\pi/2$	S_0	S_1	S_0	12	16
8-PSK	π	S_{00}	S_{10}, S_{11}	S_{00}	5.03	4
8-PSK	π	S_{00}	S_{10}	S_{10}	5.37	4
8-PSK	π	S_{00}	S_{01}	S_{00}	10.75	4
8-PSK	π	S_{00}	S_{01}	S_{01}	10.75	4
8-PSK	$\pi/2, 3\pi/2$	S_{00}	S_{10}, S_{11}	S_{00}	2.69	4
8-PSK	$\pi/2, 3\pi/2$	S_{00}	S_{10}	S_{10}	2.54	4
8-PSK	$\pi/2, 3\pi/2$	S_{00}	S_{01}	S_{01}	5.49	4
8-PSK	$\pi/2, 3\pi/2$	S_{00}	S_{01}	S_{00}	6.06	4
8-PSK	$\pi/4, 7\pi/4$	S_{00}	S_{10}, S_{11}	S_{00}	1.03	4
8-PSK	$\pi/4, 7\pi/4$	S_{00}	S_{10}	S_{10}	1.03	4
8-PSK	$\pi/4, 7\pi/4$	S_{00}	S_{01}	S_{00}	2.75	4
8-PSK	$\pi/4, 7\pi/4$	S_{00}	S_{01}	S_{01}	2.75	4
8-PSK	$3\pi/4, 5\pi/4$	S_{00}	S_{10}, S_{11}	S_{00}	4.34	4
8-PSK	$3\pi/4, 5\pi/4$	S_{00}	S_{10}	S_{10}	4.34	4
8-PSK	$3\pi/4, 5\pi/4$	S_{00}	S_{01}	S_{00}	9.37	4
8-PSK	$3\pi/4, 5\pi/4$	S_{00}	S_{01}	S_{01}	9.37	4

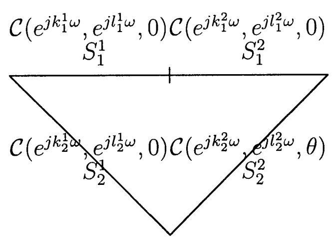

Fig. 12. Two typical paths differing in $P = 2$ transitions.

and $l_2^1 = l_2^3 = 1$ in (12), we have $\det(A) = 48$ which means

$$\min \det(A) \leq 48. \tag{20}$$

Combining inequalities (19) and (20) shows

$$\min \det(A) = 48 \tag{21}$$

which is greater than 16. □

Using the preceding result, one can calculate the coding gain of the codes in Fig. 5. It is easy to show that the minimum value of the CGD when $P > 3$ is greater than the minimum value of the CGD when $P = 3$. This proves that the minimum CGD for the code in Fig. 5 is dominated by parallel transitions and is equal to 64 for BPSK. Therefore, the coding gain is 8 for $r = 1$ bit/s/Hz using BPSK. Also, the coding gain of the code in Fig. 5 for $r = 2$ bits/se/Hz using QPSK is 4. This is due to the fact that the minimum CGD for this code is dominated by parallel transitions and is equal to 16. Therefore, the coding gain is 4.

Note that we have considered specific codes for clarity of the presentation. One can show similar results for any trellis for which it is impossible to diverge from a state and merge to the same state in $P = 2$ transitions. For the trellises that contain a path with $P = 2$ transitions (Fig. 12), we provide similar formulas in the sequel.

B. Error Events With Path Length of Two

We consider two codewords diverging from state zero and re-emerging after $P = 2$ transitions to state zero in Fig. 12. For parallel transitions ($P = 1$), one can calculate the CGD from (9). We consider a typical case where the first codeword stays at state zero. For the second codeword, the first transition diverging from state zero uses $C(x_1, x_2, 0)$ and the second transition reemerging to state zero uses $C(x_1, x_2, \theta)$ as in Fig. 12. Note that $\theta = \pi/2, \pi, 3\pi/2$ in Fig. 6, $\theta = \pi$ in Figs. 7 and 8, and $\theta = \pi/2$ in Fig. 9. It can be shown that

$$\det(A) = (a+b_1)(a+b_2) - d = a^2 + a(b_1+b_2) + b_1 b_2 - d \tag{22}$$

where

$$\begin{aligned} a &= 4 - 2\cos\left[\omega(k_2^1 - k_1^1)\right] - 2\cos\left[\omega(l_2^1 - l_1^1)\right] \\ b_1 &= 4 - 2\cos\left[\omega(k_2^2 - k_1^2) + \theta\right] - 2\cos\left[\omega(l_2^2 - l_1^2)\right] \\ b_2 &= 4 - 2\cos\left[\omega(k_2^2 - k_1^2)\right] - 2\cos\left[\omega(l_2^2 - l_1^2) - \theta\right] \\ d &= (2 - 2\cos\theta)\left(2 - 2\cos\left[\omega(k_2^2 - k_1^2 + l_1^2 - l_2^2) + \theta\right]\right) \end{aligned} \tag{23}$$

and $a, b_1, b_2, d \geq 0$. Since a only depends on $(k_2^1, k_1^1, l_2^1, l_1^1)$, $b_1 + b_2$ is not negative, and b_1, b_2, d are independent of $(k_2^1, k_1^1, l_2^1, l_1^1)$, one can first calculate $(\min a)$ and then calculate $\min \det(A)$. In other words, one can use the following formula to find $\min \det(A)$:

$$\min \det(A) = (\min a)^2 + \min[(\min a)(b_1+b_2) + b_1 b_2 - d]. \tag{24}$$

We use (24) to calculate the CGD for trellises with $P = 2$. We tabulate $\min \det(A)$ for different constellations and rotations (θ) in Table II. In Table II, we use the notation of Fig. 12, where with a slight abuse of the notation $(k_1^1, l_1^1) \in S_1^1$, $(k_1^2, l_1^2) \in S_1^2$, $(k_2^1, l_2^1) \in S_2^1$, and $(k_2^2, l_2^2) \in S_2^2$. We use these values to

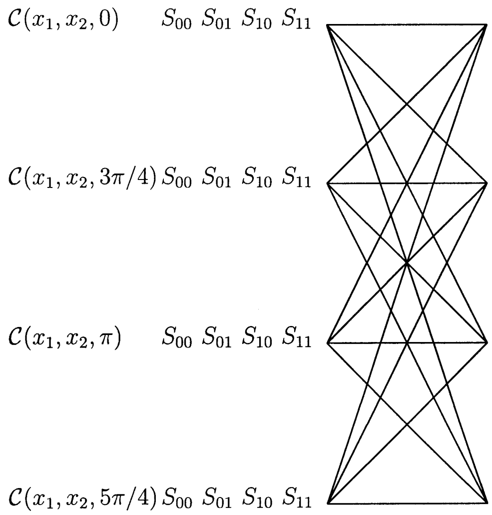

Fig. 13. A four-state code; $r = 3$ bits/s/Hz (8-PSK).

calculate the CGD of super-orthogonal space–time trellis codes. Again we consider specific examples for the sake of simplicity.

First, we calculate the coding gain for the BPSK code in Fig. 7. We assume two codewords diverging from state zero and re-emerging after P transitions to state zero. For parallel transitions ($P = 1$), one can calculate the CGD from (9) which is 64. For $P = 2$, $\min a = 4$ and

$$\min \det(A) = 16 + \min [4(b_1 + b_2) + b_1 b_2 - d] = 48 < 64. \tag{25}$$

Therefore, CGD $= 48$ and the coding gain is dominated by paths with $P = 2$ transitions and is equal to 6.92.

For the QPSK code in Fig. 7, the CGD for parallel transitions $(P = 1)$ is 16. For $P = 2$, since $\min a = 2$, we have

$$\min \det(A) = 4 + \min [2(b_1 + b_2) + b_1 b_2 - d] = 20 > 16. \tag{26}$$

Therefore, the coding gain for the QPSK code in Fig. 7 is 4.

In the case of the 8-PSK codes in Figs. 8 and 9, the parallel transitions are dominant and the coding gain is 2 (CGD $= 4$). This can be shown using (24) and Table II.

The coding gain in Fig. 6 is dominated by paths with $P = 2$ transitions. Using (24) and Table II, one can show that the minimum CGD is 2.69 as shown in Table I. We emphasis the importance of picking the right set of rotations and subsets in providing the maximum coding gain. We also note that the minimum CGD is not always enough in comparing two codes with each other. As an example, let us consider the four-state, rate 3-bits/s/Hz code in Fig. 13. We have picked $\theta = 3\pi/4,\ 5\pi/4$ for states one and three in Fig. 13 instead of $\theta = \pi/2,\ 3\pi/2$ in Fig. 6. The code in Fig. 13 is an interesting example where starting from state zero, parallel transitions are dominant and the CGD is 4. However, one should consider other error events with length two where the parallel transitions are not dominant. For example, starting from state one and staying at state one for two transitions, an error event with $P = 2$ can be a path diverging from state one to state two and re-emerging to state one in the second transition. Note that this error event can be part of two codewords with length four as illustrated in Fig. 14. The CGD for such a path is 1.03 which is lower than 2.69 and 4. There are other paths with $P = 2$ providing different CGDs. Therefore, for the code in Fig. 13, the minimum CGD is not a good indicator of the performance. In addition to the minimum CGD, one needs to consider the path weights (multiplicity) of error events. In this case, study of the distance spectrum is required to find the best code, i.e., the best assignment of rotations and subsets. Also note that this is not a linear code. While we provide a few examples, a complete study to find the best codes for each trellis and rate is beyond the scope of this paper and is left for future research.

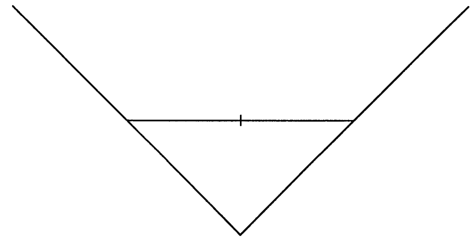

Fig. 14. Two paths with length four differing in $P = 2$ transitions.

V. Extension to More Than Two Antennas

In this section, we extend our general approach for designing super-orthogonal space–time trellis codes (SOSTTCs) to more than two transmit antennas. We follow the same principles to systematically design SOSTTCs using orthogonal designs, constellation rotations, and set partitioning. We provide a few examples to show how our general approach works.

A. Real Constellations

A full-rate real $N \times N$ orthogonal design only exists for $N = 2,\ 4,\ 8$ [8]. An example of a 4×4 real orthogonal design from [8] is given as follows:

$$\mathcal{C}(x_1,\, x_2,\, x_3,\, x_4) = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix}. \tag{27}$$

To expand the orthogonal matrices, similar to the case of two antennas in (6), we use phase rotations as follows:

$$\mathcal{C}(x_1,\, x_2,\, x_3,\, x_4,\, \theta_1,\, \theta_2,\, \theta_3) = \begin{pmatrix} x_1 e^{j\theta_1} & x_2 e^{j\theta_2} & x_3 e^{j\theta_3} & x_4 \\ -x_2 e^{j\theta_1} & x_1 e^{j\theta_2} & -x_4 e^{j\theta_3} & x_3 \\ -x_3 e^{j\theta_1} & x_4 e^{j\theta_2} & x_1 e^{j\theta_3} & -x_2 \\ -x_4 e^{j\theta_1} & -x_3 e^{j\theta_2} & x_2 e^{j\theta_3} & x_1 \end{pmatrix}. \tag{28}$$

Since the constellation is real (for example, BPSK or PAM) and we do not want to expand the constellation signals, we pick $\theta_i = 0,\ \pi$, where $i = 1,\ 2,\ 3$. This means that we only potentially use a sign change for each column. Note that, in general, for N transmit antennas, one can use $N - 1$ rotations. Equation (27) is a member of the super-orthogonal sets in (28) for $\theta_1 = \theta_2 = \theta_3 = 0$. In other words, with a slight abuse of the notation, we have $\mathcal{C}(x_1,\, x_2,\, x_3,\, x_4) = \mathcal{C}(x_1,\, x_2,\, x_3,\, x_4,\, 0,\, 0,\, 0)$.

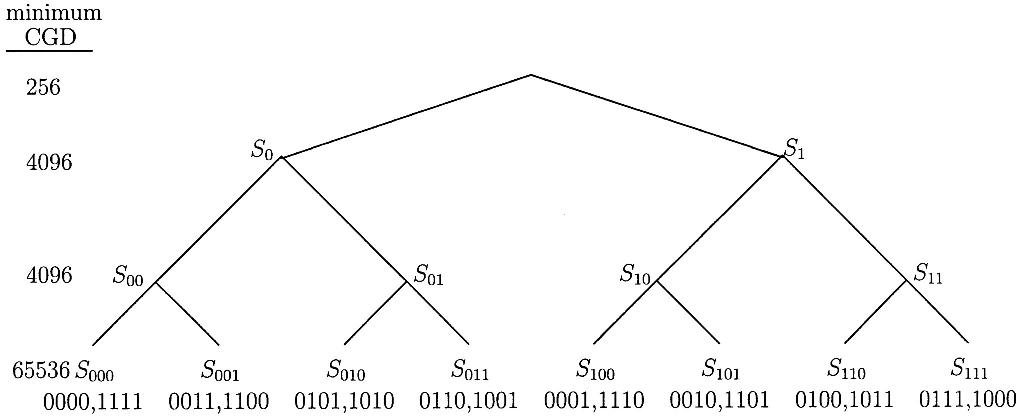

Fig. 15. Set partitioning for four transmit antennas (BPSK); the numbers at leaves represent the indexes of the symbols in the space–time block code.

Similar to the case of two transmit antennas, we provide a set partitioning for super-orthogonal codes and show how to maximize the coding gain without sacrificing the rate. Following the definitions in [1], for a full-diversity code, the minimum of the determinant of the matrix $A(c_1, c_2) = B(c_1, c_2)B^H(c_1, c_2)$ over all possible pairs of distinct codewords c_1 and c_2 corresponds to the coding gain. Similar to the case of two transmit antennas, the CGD between codewords c_1 and c_2 is defined as $d^2(c_1, c_2) = \det(A(c_1, c_2))$, where $\det(A)$ is the determinant of matrix A. Then, we use the CGD instead of Euclidean distance to define a set partitioning similar to Ungerboeck's set partitioning [20]. An example for a BPSK constellation is shown in Fig. 15. At the root of the tree, the minimum determinant is 256. At the first level of partitioning, the highest determinant that can be obtained is 4096, which is obtained by creating subsets S_0 and S_1 with transmitted signal elements differing at least in two positions. At the last level of partitioning, we have eight sets S_{000}, S_{001}, S_{010}, S_{011}, S_{100}, S_{101}, S_{110}, and S_{111} with two elements per set that differ in all positions. The resulting minimum CGD is 65536.

Code construction based on super-orthogonal sets is as follows. We assign a constituent space–time block code to all transitions from a state. The adjacent states are typically assigned to one of the other constituent space–time block code from the super-orthogonal code. Similarly, we can assign the same space–time block code to branches that are merging into a state. It is thus assured that any path that diverges from (or merges to) the correct path differs by rank 4. In other words, every pair of codewords diverging from (or merging to) a state achieves full diversity because the pair is from the same orthogonal code (same parameters θ_1, θ_2, θ_3). On the other hand, for codewords with different θ_1, θ_2, θ_3, it is possible that they do not achieve full diversity. Since these codewords are assigned to different states, the resulting trellis code would provide full diversity despite the fact that a pair of codewords in a super-orthogonal code may not achieve full diversity.

Figs. 16–18 show the examples of two-state, four-state, and eight-state codes for transmitting $r = 1$ bit/s/Hz using BPSK, respectively.

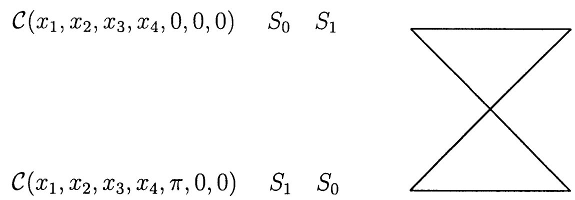

Fig. 16. A two-state code; four transmit antennas; $r = 1$ bit/s/Hz (BPSK).

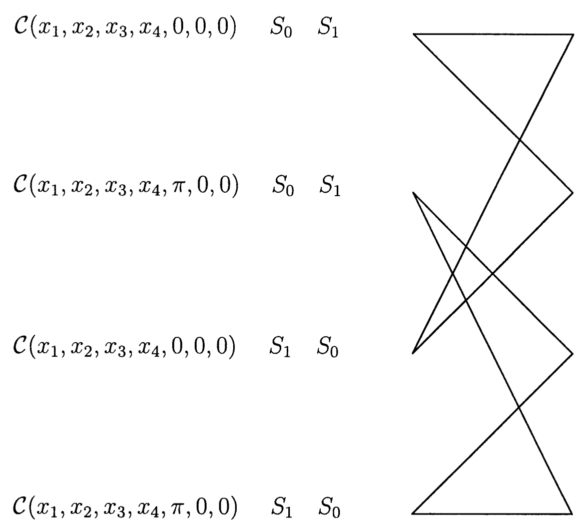

Fig. 17. A four-state code; four transmit antennas; $r = 1$ bit/s/Hz (BPSK).

B. Complex Constellations

A full-rate complex orthogonal design does not exist for more than two transmit antennas [8]. A rate $3/4$ complex orthogonal design for four transmit antennas was proposed in [8]. It has been shown in [22] that this is the maximum possible rate for four transmit antennas. An example of a 4×4 rate $3/4$ complex orthogonal design is given as follows:

$$\mathcal{C}(x_1, x_2, x_3) = \begin{pmatrix} x_1 & x_2 & x_3 & 0 \\ -x_2^* & x_1^* & 0 & -x_3 \\ x_3^* & 0 & -x_1^* & -x_2 \\ 0 & -x_3^* & x_2^* & -x_1 \end{pmatrix}. \quad (29)$$

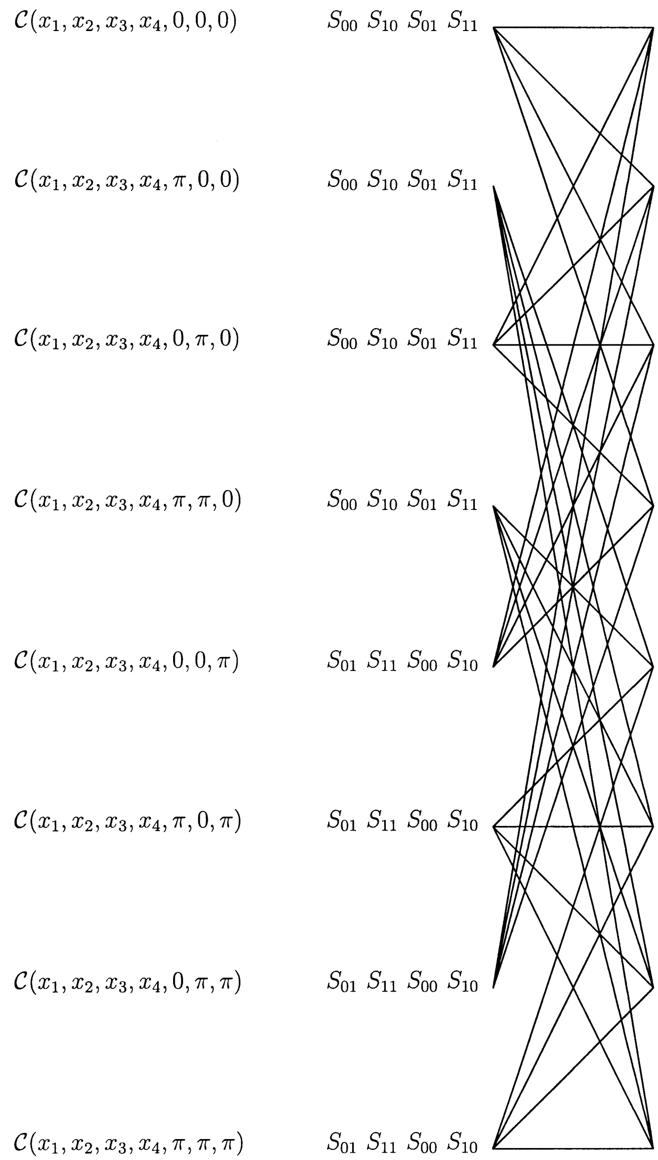

Fig. 18. An eight-state code; four transmit antennas; $r = 1$ bit/s/Hz (BPSK).

To expand the orthogonal matrices, similar to the case of real constellations, we use the following rotations:

$$\mathcal{C}(x_1, x_2, x_3, \theta_1, \theta_2, \theta_3) = \begin{pmatrix} x_1 e^{j\theta_1} & x_2 e^{j\theta_2} & x_3 e^{j\theta_3} & 0 \\ -x_2^* e^{j\theta_1} & x_1^* e^{j\theta_2} & 0 & -x_3 \\ x_3^* e^{j\theta_1} & 0 & -x_1^* e^{j\theta_3} & -x_2 \\ 0 & -x_3^* e^{j\theta_2} & x_2^* e^{j\theta_3} & -x_1 \end{pmatrix}. \quad (30)$$

Again, we only use rotations that do not expand the constellation. For example, $\theta_1, \theta_2, \theta_3$ can be $0, \pi/2, \pi, 3\pi/2$ for QPSK. Then, similar to the case of real constellations and two transmit antennas, after set partitioning we can systematically design SOSTTCs for any trellis and complex constellation.

VI. Simulation Results

In this section, we provide simulation results for our new SOSTTCs using two transmit antennas and one receive antenna. We compare our results with those of the existing space-time trellis codes in the literature when a comparable code exists. In all simulations, similar to the results in [1], a frame consists of 130 transmissions out of each transmit antenna.

Fig. 19 shows the frame error probability results versus SNR for the codes in Figs. 5 and 7 using BPSK and the corresponding set partitioning in Fig. 1. Both of these codes are full-rate and transmit 1 bit/s/Hz. Note that one cannot design such a code using the scheme developed in [10] and [11].

Fig. 20 shows the simulation results for transmitting 2 bits/s/Hz using a QPSK constellation. The codes in Figs. 5 and 7 are denoted by "SOSTTC 4 states" and "SOSTTC 2 states," respectively. The corresponding results for a code with the same rate and four states from [1] is also provided ("STTC") for comparison. We also include the results for four-state codes form [3] ("YB"), [4] ("BBH"), and [5] ("CYV"). To the best of our knowledge, these are the best available codes in the literature. Note that while the codes in [3]–[5] provide some improvements over the original space–time trellis codes in [1] for more than one receive antennas, their performance is very close to that of [1] for the case of one receive antenna. As can be seen from the figure, our four-state super-orthogonal space–time trellis code outperforms the corresponding space–time trellis codes by more than 2 dB. The performance of our four-state SOSTTC outperforms that of a 32-state space–time trellis codes in [1] and is very close to that of a 64-state space–time trellis code in [1]. We also note that our SOSTTC preserves the decoding simplicity of orthogonal designs (with added complexity for the Viterbi algorithm). In fact, when redundant computations are avoided and the orthogonality of the building blocks is smartly used, the decoding complexity of a SOSTTC is less than that of a space–time trllis code using the same trellis. The details of simple decoding are explained in [14], [15] and we do not repeat them here.

VII. Conclusion

We have introduced a new structure for designing space–time trellis codes which is called super-orthogonal space-time trellis codes (SOSTTCs). While providing full diversity and full rate, the structure of our new codes allows an increase in the coding gain. Not only does our SOSTTC outperform the space-time trellis codes in the literature, but it also provides a systematic method for designing space–time trellis codes at different rates and for different trellises. Since we have used orthogonal designs as the building blocks in our SOSTTCs, the complexity of the decoding remains low while full diversity is guaranteed. Codes operating at different rates, up to the highest theoretically possible rate, for different number of states, can be designed by using our optimal set partitioning. In general, SOSTTCs can provide a tradeoff between rate and coding gain while achieving full diversity.

Our code design strategy is general enough such that a quasi-orthogonal space–time block code [23] or any other structure which guarantees diversity is applicable as well. Although our general approach is applicable to other nonorthogonal coding structures, there are many details that are left for future research.

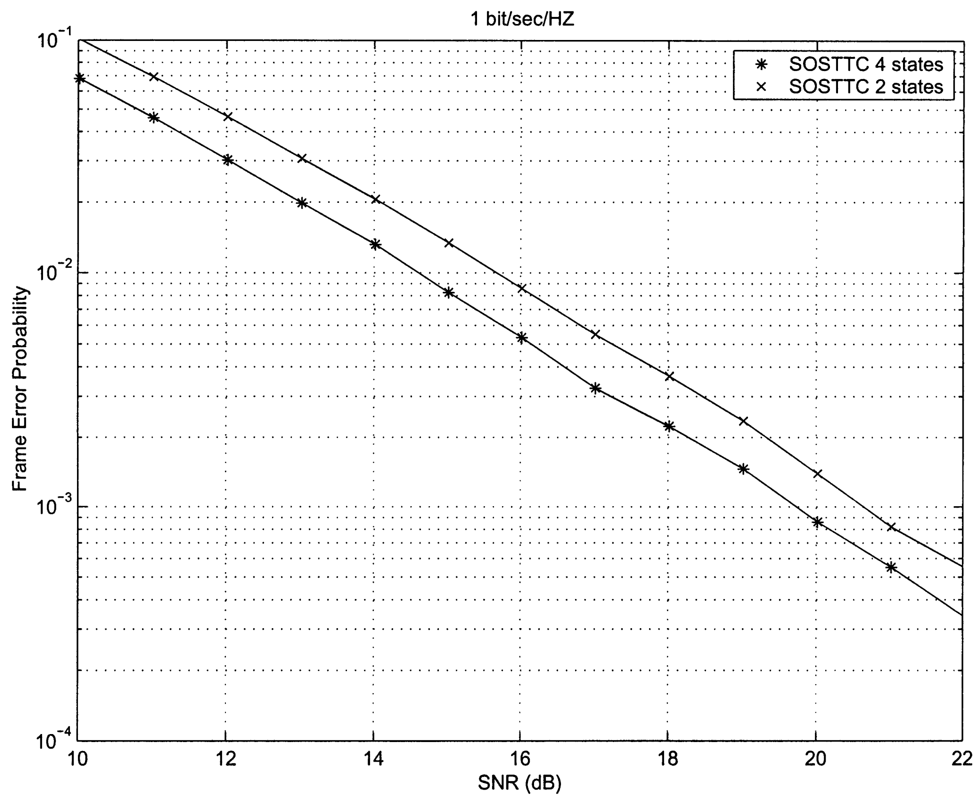

Fig. 19. Simulation results for $r = 1$ bit/s/Hz (BPSK).

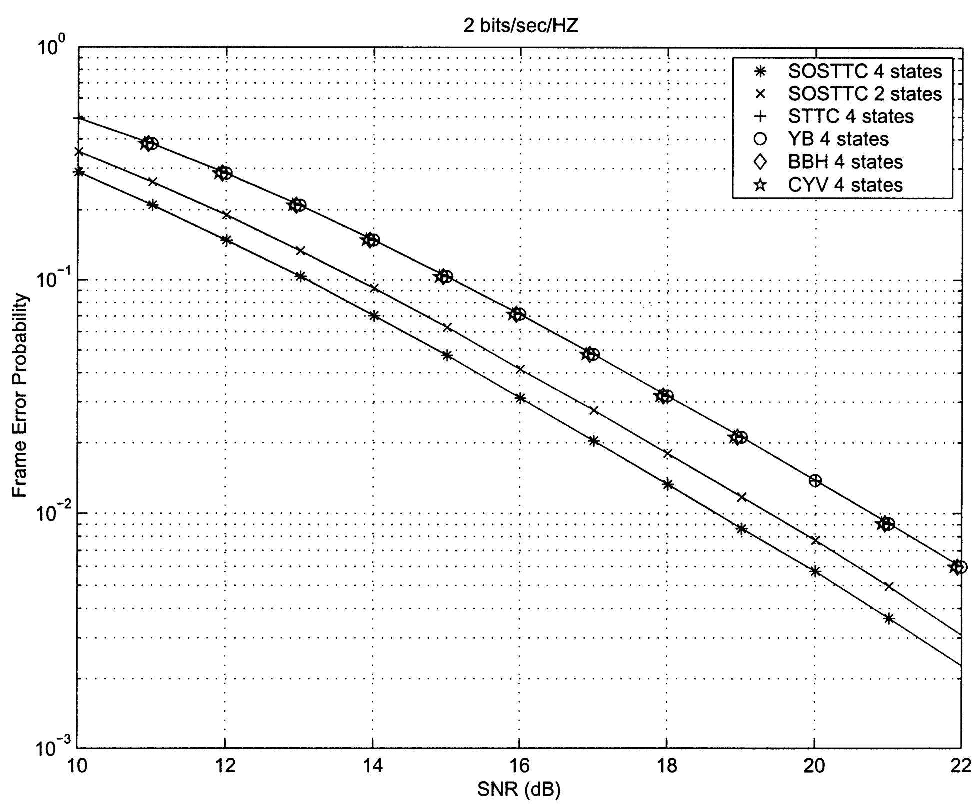

Fig. 20. Simulation results for $r = 2$ bits/s/Hz (QPSK).

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